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Balancing of Reciprocating Masses

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22.1. Introduction

We have discussed in Chapter 15 (Art. 15.10), the various forces acting on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as **unbalanced force** or **shaking force**. Thus if the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present.

Consider a horizontal reciprocating engine mechanism as shown in Fig. 22.1.

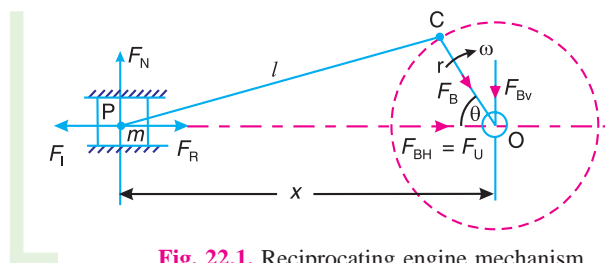


Fig. 22.1. Reciprocating engine mechanism.

Let F_R = Force required to accelerate the reciprocating parts,

F_I = Inertia force due to reciprocating parts,

F_N = Force on the sides of the cylinder walls or normal force acting on the cross-head guides, and

F_B = Force acting on the crankshaft bearing or main bearing.

Since F_R and F_I are equal in magnitude but opposite in direction, therefore they balance each other. The horizontal component of F_B (i.e. F_{BH}) acting along the line of reciprocation is also equal and opposite to F_I . This force $F_{BH} = F_U$ is an unbalanced force or shaking force and required to be properly balanced.

The force on the sides of the cylinder walls (F_N) and the vertical component of F_B (i.e. F_{BV}) are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ or $F_{BV} \times x$.

From above we see that the effect of the reciprocating parts is to produce a shaking force and a shaking couple. Since the shaking force and a shaking couple vary in magnitude and direction during the engine cycle, therefore they cause very objectionable vibrations.

Thus the purpose of balancing the reciprocating masses is to eliminate the shaking force and a shaking couple. In most of the mechanisms, we can reduce the shaking force and a shaking couple by adding appropriate balancing mass, but it is usually not practical to eliminate them completely. In other words, the reciprocating masses are only partially balanced.

Note : The masses rotating with the crankshaft are normally balanced and they do not transmit any unbalanced or shaking force on the body of the engine.

22.2. Primary and Secondary Unbalanced Forces of Reciprocating Masses

Consider a reciprocating engine mechanism as shown in Fig. 22.1.

Let m = Mass of the reciprocating parts,

l = Length of the connecting rod PC ,

r = Radius of the crank OC ,

θ = Angle of inclination of the crank with the line of stroke PO ,

ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r .

We have already discussed in Art. 15.8 that the acceleration of the reciprocating parts is approximately given by the expression,

$$a_R = \omega^2 \cdot r \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

\therefore Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

We have discussed in the previous article that the horizontal component of the force exerted on the crank shaft bearing (i.e. F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U .

\therefore Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r \left(\cos \theta + \frac{\cos 2\theta}{n} \right) = m \cdot \omega^2 \cdot r \cos \theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r \cos \theta)$ is known as **primary unbalanced force** and

$\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} \right)$ is called **secondary unbalanced force**.

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∴ Primary unbalanced force, $F_P = m \cdot \omega^2 \cdot r \cos \theta$

and secondary unbalanced force, $F_S = m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}$

Notes: 1. The primary unbalanced force is maximum, when $\theta = 0^\circ$ or 180° . Thus, the primary force is maximum twice in one revolution of the crank. The maximum primary unbalanced force is given by

$$F_{P(max)} = m \cdot \omega^2 \cdot r$$

2. The secondary unbalanced force is maximum, when $\theta = 0^\circ, 90^\circ, 180^\circ$ and 360° . Thus, the secondary force is maximum four times in one revolution of the crank. The maximum secondary unbalanced force is given by

$$F_{S(max)} = m \cdot \omega^2 \times \frac{r}{n}$$

3. From above we see that secondary unbalanced force is $1/n$ times the maximum primary unbalanced force.

4. In case of moderate speeds, the secondary unbalanced force is so small that it may be neglected as compared to primary unbalanced force.

5. The unbalanced force due to reciprocating masses varies in magnitude but constant in direction while due to the revolving masses, the unbalanced force is constant in magnitude but varies in direction.

22.3. Partial Balancing of Unbalanced Primary Force In a Reciprocating Engine

The primary unbalanced force ($m \cdot \omega^2 \cdot r \cos \theta$) may be considered as the component of the centrifugal force produced by a rotating mass m placed at the crank radius r , as shown in Fig. 22.2.

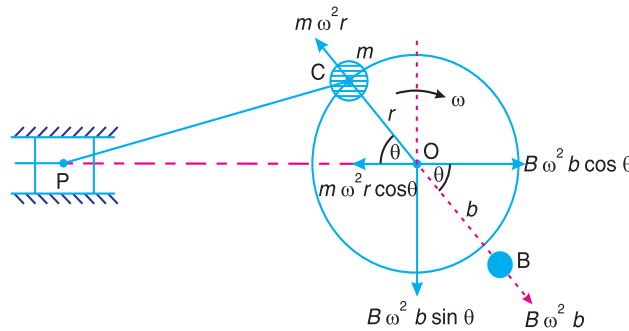


Fig. 22.2. Partial balancing of unbalanced primary force in a reciprocating engine.

The primary force acts from O to P along the line of stroke. Hence, balancing of primary force is considered as equivalent to the balancing of mass m rotating at the crank radius r . This is balanced by having a mass B at a radius b , placed diametrically opposite to the crank pin C .

We know that centrifugal force due to mass B ,

$$= B \cdot \omega^2 \cdot b$$

and horizontal component of this force acting in opposite direction of primary force

$$= B \cdot \omega^2 \cdot b \cos \theta$$

The primary force is balanced, if

$$B \cdot \omega^2 \cdot b \cos \theta = m \cdot \omega^2 \cdot r \cos \theta \quad \text{or} \quad B \cdot b = m \cdot r$$

A little consideration will show, that the primary force is completely balanced if $B.b = m.r$, but the centrifugal force produced due to the revolving mass B , has also a vertical component (perpendicular to the line of stroke) of magnitude $B \cdot \omega^2 \cdot b \sin \theta$. This force remains unbalanced. The maximum value of this force is equal to $B \cdot \omega^2 \cdot b$ when θ is 90° and 270° , which is same as the maximum value of the primary force $m \cdot \omega^2 \cdot r$.

From the above discussion, we see that in the first case, the primary unbalanced force acts along the line of stroke whereas in the second case, the unbalanced force acts along the perpendicular to the line of stroke. The maximum value of the force remains same in both the cases. It is thus obvious, that the effect of the above method of balancing is to change the direction of the maximum unbalanced force from the line of stroke to the perpendicular of line of stroke. As a compromise let a fraction 'c' of the reciprocating masses is balanced, such that

$$c.m.r = B.b$$

∴ Unbalanced force along the line of stroke

$$\begin{aligned} &= m \cdot \omega^2 \cdot r \cos \theta - B \cdot \omega^2 \cdot b \cos \theta \\ &= m \cdot \omega^2 \cdot r \cos \theta - c \cdot m \cdot \omega^2 \cdot r \cos \theta && \dots (\because B.b = c.m.r) \\ &= (1-c)m \cdot \omega^2 \cdot r \cos \theta \end{aligned}$$

and unbalanced force along the perpendicular to the line of stroke

$$= B \cdot \omega^2 \cdot b \sin \theta = c \cdot m \cdot \omega^2 \cdot r \sin \theta$$

∴ Resultant unbalanced force at any instant

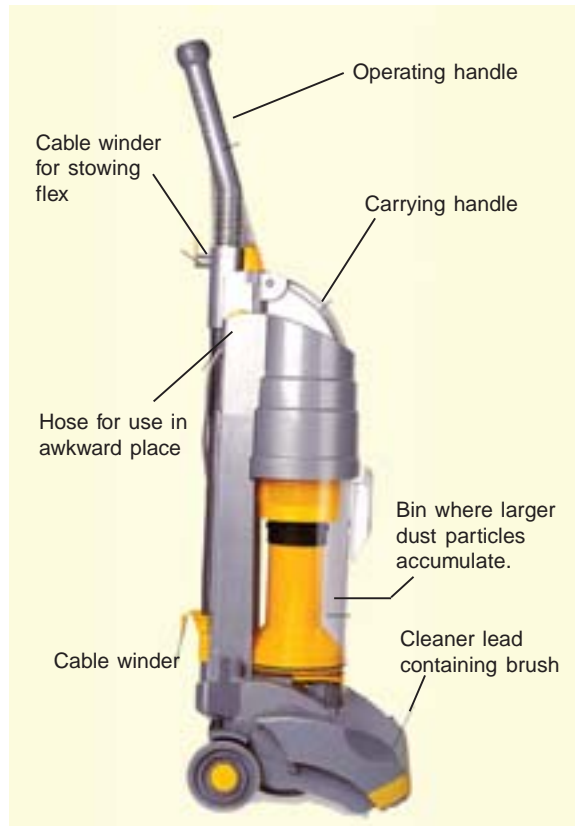
$$\begin{aligned} &= \sqrt{\left[(1-c)m \cdot \omega^2 \cdot r \cos \theta \right]^2 + \left[c \cdot m \cdot \omega^2 \cdot r \sin \theta \right]^2} \\ &= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2 \theta + c^2 \sin^2 \theta} \end{aligned}$$

Note : If the balancing mass is required to balance the revolving masses as well as reciprocating masses, then

$$B.b = m_1 \cdot r + c \cdot m \cdot r = (m_1 + c \cdot m)r$$

where

m_1 = Magnitude of the revolving masses, and
 m = magnitude of the reciprocating masses.



Cyclone cleaner.

Example 22.1. A single cylinder reciprocating engine has speed 240 r.p.m., stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 37 kg. If two-third of the reciprocating parts and all the revolving parts are to be balanced, find : **1.** The balance mass required at a radius of 400 mm, and **2.** The residual unbalanced force when the crank has rotated 60° from top dead centre.

Solution. Given : $N = 240$ r.p.m. or $\omega = 2\pi \times 240/60 = 25.14$ rad/s ; Stroke = 300 mm = 0.3 m ; $m = 50$ kg ; $m_1 = 37$ kg ; $r = 150$ mm = 0.15 m ; $c = 2/3$

1. Balance mass required

Let $B =$ Balance mass required, and
 $b =$ Radius of rotation of the balance mass = 400 mm = 0.4 m
 . . . (Given)

We know that

$$B \cdot b = (m_1 + c \cdot m) r$$

$$B \times 0.4 = \left(37 + \frac{2}{3} \times 50 \right) 0.15 = 10.55 \quad \text{or} \quad B = 26.38 \text{ kg Ans.}$$

2. Residual unbalanced force

Let $\theta =$ Crank angle from top dead centre = 60° . . . (Given)

We know that residual unbalanced force

$$= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2 \theta + c^2 \sin^2 \theta}$$

$$= 50(25.14)^2 \cdot 0.15 \sqrt{\left(1 - \frac{2}{3}\right)^2 \cos^2 60^\circ + \left(\frac{2}{3}\right)^2 \sin^2 60^\circ} \text{ N}$$

$$= 4740 \times 0.601 = 2849 \text{ N Ans.}$$

22.4. Partial Balancing of Locomotives

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives may be classified as :

1. Inside cylinder locomotives ; and
2. Outside cylinder locomotives.

In the **inside cylinder locomotives**, the two cylinders are placed in between the planes of two driving wheels as shown in Fig. 22.3 (a) ; whereas in the **outside cylinder locomotives**, the two cylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown in Fig. 22.3 (b). The locomotives may be

- (a) Single or uncoupled locomotives ; and
- (b) Coupled locomotives.

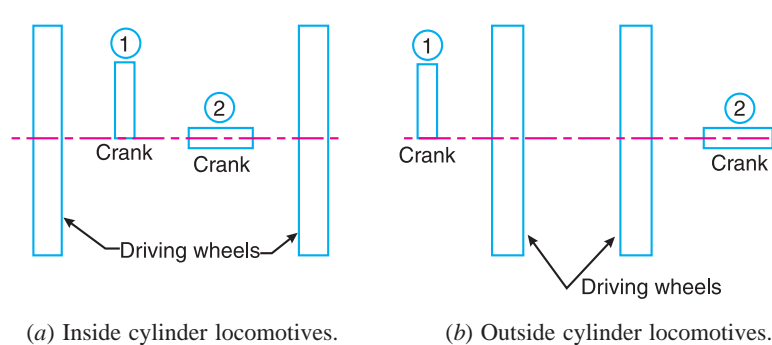


Fig. 22.3

A *single* or *uncoupled locomotive* is one, in which the effort is transmitted to one pair of the wheels only ; whereas in *coupled locomotives*, the driving wheels are connected to the leading and trailing wheel by an outside coupling rod.

22.5. Effect of Partial Balancing of Reciprocating Parts of Two Cylinder Locomotives

We have discussed in the previous article that the reciprocating parts are only partially balanced. Due to this partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce;

1. Variation in tractive force along the line of stroke ; and
2. Swaying couple.

The effect of an unbalanced primary force perpendicular to the line of stroke is to produce variation in pressure on the rails, which results in hammering action on the rails. The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as a *hammer blow*. We shall now discuss the effects of an unbalanced primary force in the following articles.

22.6. Variation of Tractive Force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as *tractive force*. Let the crank for the first cylinder be inclined at an angle θ with the line of stroke, as shown in Fig. 22.4. Since the crank for the second cylinder is at right angle to the first crank, therefore the angle of inclination for the second crank will be $(90^\circ + \theta)$.

- Let m = Mass of the reciprocating parts per cylinder, and
 c = Fraction of the reciprocating parts to be balanced.

We know that unbalanced force along the line of stroke for cylinder 1

$$= (1-c)m.\omega^2.r \cos \theta$$

Similarly, unbalanced force along the line of stroke for cylinder 2,

$$= (1-c)m.\omega^2.r \cos(90^\circ + \theta)$$

∴ As per definition, the tractive force,

$$F_T = \text{Resultant unbalanced force along the line of stroke}$$

$$= (1-c)m.\omega^2.r \cos \theta + (1-c)m.\omega^2.r \cos(90^\circ + \theta) = (1-c)m.\omega^2.r(\cos \theta - \sin \theta)$$

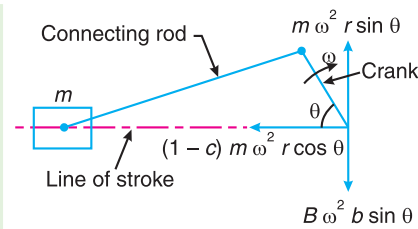


Fig. 22.4. Variation of tractive force.

The tractive force is maximum or minimum when $(\cos \theta - \sin \theta)$ is maximum or minimum. For $(\cos \theta - \sin \theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos \theta - \sin \theta) = 0 \quad \text{or} \quad -\sin \theta - \cos \theta = 0 \quad \text{or} \quad -\sin \theta = \cos \theta$$

$$\therefore \tan \theta = -1 \quad \text{or} \quad \theta = 135^\circ \quad \text{or} \quad 315^\circ$$

Thus, the tractive force is maximum or minimum when $\theta = 135^\circ$ or 315° .

∴ Maximum and minimum value of the tractive force or the variation in tractive force

$$= \pm(1-c)m.\omega^2.r(\cos 135^\circ - \sin 135^\circ) = \pm\sqrt{2}(1-c)m.\omega^2.r$$

22.7. Swaying Couple

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line YY between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as **swaying couple**.

Let $a =$ Distance between the centre lines of the two cylinders.

∴ Swaying couple

$$\begin{aligned}
 &= (1-c)m\omega^2.r \cos \theta \times \frac{a}{2} \\
 &\quad - (1-c)m\omega^2.r \cos (90^\circ + \theta) \frac{a}{2} \\
 &= (1-c)m\omega^2.r \times \frac{a}{2} (\cos \theta + \sin \theta)
 \end{aligned}$$

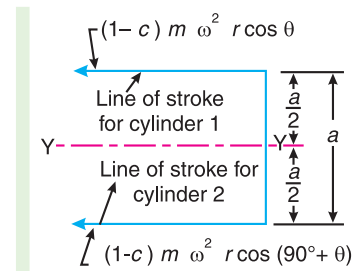


Fig. 22.5. Swaying couple.

The swaying couple is maximum or minimum when $(\cos \theta + \sin \theta)$ is maximum or minimum. For $(\cos \theta + \sin \theta)$ to be maximum or minimum,

$$\frac{d}{d\theta} (\cos \theta + \sin \theta) = 0 \quad \text{or} \quad -\sin \theta + \cos \theta = 0 \quad \text{or} \quad -\sin \theta = -\cos \theta$$

$$\therefore \quad \tan \theta = 1 \quad \text{or} \quad \theta = 45^\circ \quad \text{or} \quad 225^\circ$$

Thus, the swaying couple is maximum or minimum when $\theta = 45^\circ$ or 225° .

∴ Maximum and minimum value of the swaying couple

$$= \pm (1-c)m\omega^2.r \times \frac{a}{2} (\cos 45^\circ + \sin 45^\circ) = \pm \frac{a}{\sqrt{2}} (1-c)m\omega^2.r$$

Note : In order to reduce the magnitude of the swaying couple, revolving balancing masses are introduced. But, as discussed in the previous article, the revolving balancing masses cause unbalanced forces to act at right angles to the line of stroke. These forces vary the downward pressure of the wheels on the rails and cause oscillation of the locomotive in a vertical plane about a horizontal axis. Since a swaying couple is more harmful than an oscillating couple, therefore a value of 'c' from 2/3 to 3/4, in two-cylinder locomotives with two pairs of coupled wheels, is usually used. But in large four cylinder locomotives with three or more pairs of coupled wheels, the value of 'c' is taken as 2/5.

22.8. Hammer Blow

We have already discussed that the maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as **hammer blow**.

We know that the unbalanced force along the perpendicular to the line of stroke due to the balancing mass B , at a radius b , in order to balance reciprocating parts only is $B.\omega^2.b \sin \theta$. This force will be maximum when $\sin \theta$ is unity, i.e. when $\theta = 90^\circ$ or 270° .

$$\therefore \quad \text{Hammer blow} = B.\omega^2.b \quad \text{(Substituting } \sin \theta = 1)$$

The effect of hammer blow is to cause the variation in pressure between the wheel and the rail. This variation is shown in Fig. 22.6, for one revolution of the wheel.

Let P be the downward pressure on the rails (or static wheel load).

∴ Net pressure between the wheel and the rail

$$= P \pm B.\omega^2.b$$

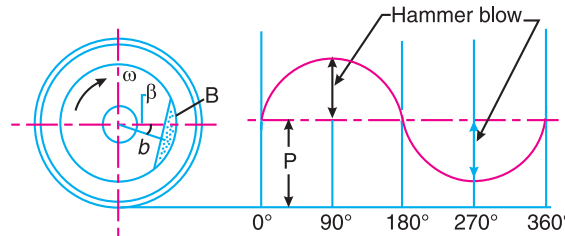


Fig. 22.6. Hammer blow.

If $(P - B.\omega^2.b)$ is **negative**, then the wheel will be lifted from the rails. Therefore the limiting condition in order that the wheel does not lift from the rails is given by

$$P = B.\omega^2.b$$

and the permissible value of the angular speed,

$$\omega = \sqrt{\frac{P}{B.b}}$$

Example 22.2. An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles.

The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m.

Solution. Given : $a = 0.7$ m; $l_B = l_C = 0.6$ m or $r_B = r_C = 0.3$ m; $m_1 = 150$ kg; $m_2 = 180$ kg; $c = 2/3$; $r_A = r_D = 0.6$ m; $N = 300$ r.p.m. or $\omega = 2\pi \times 300 / 60 = 31.42$ rad/s

We know that the equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m = m_B = m_C = m_1 + c.m_2 = 150 + \frac{2}{3} \times 180 = 270 \text{ kg}$$

Magnitude and direction of the balancing masses

Let m_A and m_D = Magnitude of the balancing masses

θ_A and θ_D = Angular position of the balancing masses m_A and m_D from the first crank B .



This Brinell hardness testing machine is used to test the hardness of the metal.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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The magnitude and direction of the balancing masses may be determined graphically as discussed below :

1. First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 22.7 (a). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder B in the horizontal direction, draw OC and OB at right angles to each other as shown in Fig. 22.7 (b).
2. Tabulate the data as given in the following table. Assume the plane of wheel A as the reference plane.

Table 22.1

Plane (1)	mass. (m) kg (2)	Radius (r)m (3)	Cent. force + ω^2 (m.r) kg-m (4)	Distance from plane A (l)m (5)	Couple + ω^2 (m.r.l) kg-m ² (6)
A (R.P.)	m_A	0.6	$0.6 m_A$	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6 m_D$	1.5	$0.9 m_D$

3. Now, draw the couple polygon from the data given in Table 22.1 (column 6), to some suitable scale, as shown in Fig 22.7 (c). The closing side $c'o'$ represents the balancing couple and it is proportional to $0.9 m_D$. Therefore, by measurement,

$$0.9 m_D = \text{vector } c'o' = 94.5 \text{ kg-m}^2 \text{ or } m_D = 105 \text{ kg Ans.}$$

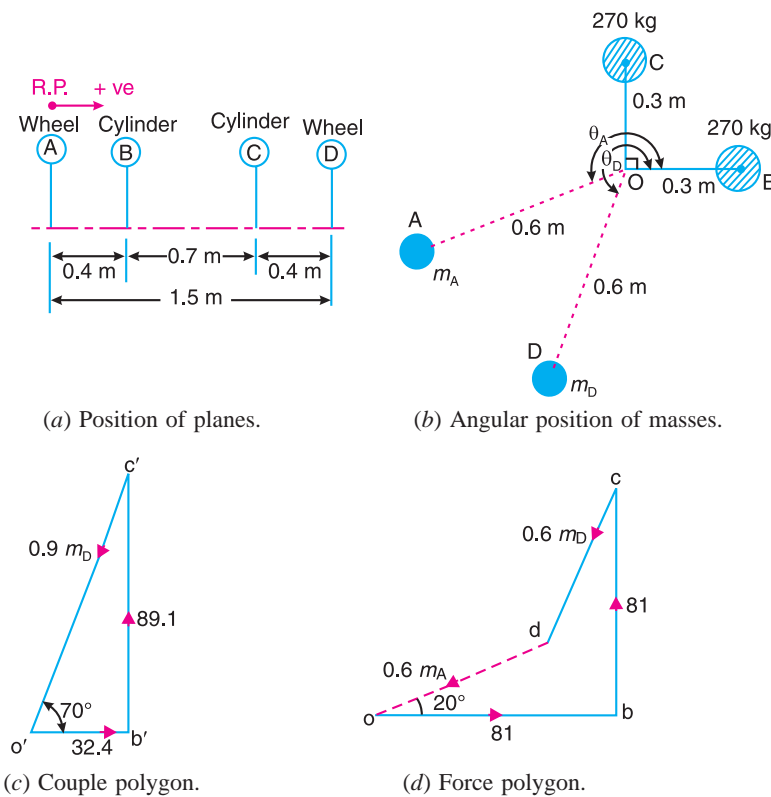


Fig. 22.7

4. To determine the angular position of the balancing mass D , draw OD in Fig. 22.7 (b) parallel to vector $c'o'$. By measurement,

$$\theta_D = 250^\circ \text{ Ans.}$$

5. In order to find the balancing mass A , draw the force polygon from the data given in Table 22.1 (column 4), to some suitable scale, as shown in Fig. 22.7 (d), The vector do represents the balancing force and it is proportional to $0.6 m_A$. Therefore by measurement,

$$0.6 m_A = \text{vector } do = 63 \text{ kg-m or } m_A = 105 \text{ kg Ans.}$$

6. To determine the angular position of the balancing mass A , draw OA in Fig. 22.7 (b) parallel to vector do . By measurement,

$$\theta_A = 200^\circ \text{ Ans.}$$

Fluctuation in rail pressure

We know that each balancing mass

$$= 105 \text{ kg}$$

∴ Balancing mass for rotating masses,

$$D = \frac{m_1}{m} \times 105 = \frac{150}{270} \times 105 = 58.3 \text{ kg}$$

and balancing mass for reciprocating masses,

$$B = \frac{c.m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6 \text{ kg}$$

This balancing mass of 46.6 kg for reciprocating masses gives rise to the centrifugal force.

∴ Fluctuation in rail pressure or hammer blow

$$= B.\omega^2.b = 46.6 (31.42)^2 0.6 = 27\ 602 \text{ N. Ans.} \quad \dots (\because b = r_A = r_D)$$

Variation of tractive effort

We know that maximum variation of tractive effort

$$= \pm \sqrt{2}(1-c)m_2.\omega^2.r = \pm \sqrt{2} \left(1 - \frac{2}{3}\right) 180(31.42)^2 0.3 \text{ N}$$

$$= \pm 25\ 127 \text{ N Ans.} \quad \dots (\because r = r_B = r_C)$$

Swaying couple

We know that maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7 \left(1 - \frac{2}{3}\right)}{\sqrt{2}} \times 180(31.42)^2 0.3 \text{ N-m}$$

$$= 8797 \text{ N-m Ans.}$$

Example 22.3 The three cranks of a three cylinder locomotive are all on the same axle and are set at 120° . The pitch of the cylinders is 1 metre and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank.

If 40% of the reciprocating parts are to be balanced, find :

1. the magnitude and the position of the balancing masses required at a radius of 0.6 m ; and
2. the hammer blow per wheel when the axle makes 6 r.p.s.

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Solution. Given : $\angle AOB = \angle BOC = \angle COA = 120^\circ$; $l_A = l_B = l_C = 0.6$ m or $r_A = r_B = r_C = 0.3$ m ; $m_1 = 300$ kg ; $m_O = 260$ kg ; $c = 40\% = 0.4$; $b_1 = b_2 = 0.6$ m ; $N = 6$ r.p.s. = $6 \times 2\pi = 37.7$ rad/s

Since 40% of the reciprocating masses are to be balanced, therefore mass of the reciprocating parts to be balanced for each outside cylinder,

$$m_A = m_C = c \times m_O = 0.4 \times 260 = 104 \text{ kg}$$

and mass of the reciprocating parts to be balanced for inside cylinder,

$$m_B = c \times m_1 = 0.4 \times 300 = 120 \text{ kg}$$

1. Magnitude and position of the balancing masses

Let B_1 and B_2 = Magnitude of the balancing masses in kg,

θ_1 and θ_2 = Angular position of the balancing masses B_1 and B_2 from crank A.

The magnitude and position of the balancing masses may be determined graphically as discussed below :

1. First of all, draw the position of planes and cranks as shown in Fig. 22.8 (a) and (b) respectively. The position of crank A is assumed in the horizontal direction.
2. Tabulate the data as given in the following table. Assume the plane of balancing mass B_1 (i.e. plane 1) as the reference plane.

Table 22.2

Plane (1)	Mass (m)kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from plane 1 (l)m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
A	104	0.3	31.2	- 0.2	- 6.24
1 (R.P.)	B_1	0.6	$0.6 B_1$	0	0
B	120	0.3	36	0.8	28.8
2	B_2	0.6	$0.6 B_2$	1.6	$0.96 B_2$
C	104	0.3	31.2	1.8	56.16

3. Now draw the couple polygon with the data given in Table 22.2 (column 6), to some suitable scale, as shown in Fig. 22.8 (c). The closing side $c'o'$ represents the balancing couple and it is proportional to $0.96 B_2$. Therefore, by measurement,

$$0.96 B_2 = \text{vector } c'o' = 55.2 \text{ kg-m}^2 \text{ or } B_2 = 57.5 \text{ kg Ans.}$$

4. To determine the angular position of the balancing mass B_2 , draw OB_2 parallel to vector $c'o'$ as shown in Fig. 22.8 (b). By measurement,

$$\theta_2 = 24^\circ \text{ Ans.}$$

5. In order to find the balance mass B_1 , draw the force polygon with the data given in Table 22.2 (column 4), to some suitable scale, as shown in Fig. 22.8 (d). The closing side co represents the balancing force and it is proportional to $0.6 B_1$. Therefore, by measurement,

$$0.6 B_1 = \text{vector } co = 34.5 \text{ kg-m or } B_1 = 57.5 \text{ kg Ans.}$$

6. To determine the angular position of the balancing mass B_1 , draw OB_1 parallel to vector co , as shown in Fig. 22.8 (b). By measurement,

$$\theta_1 = 215^\circ \text{ Ans.}$$

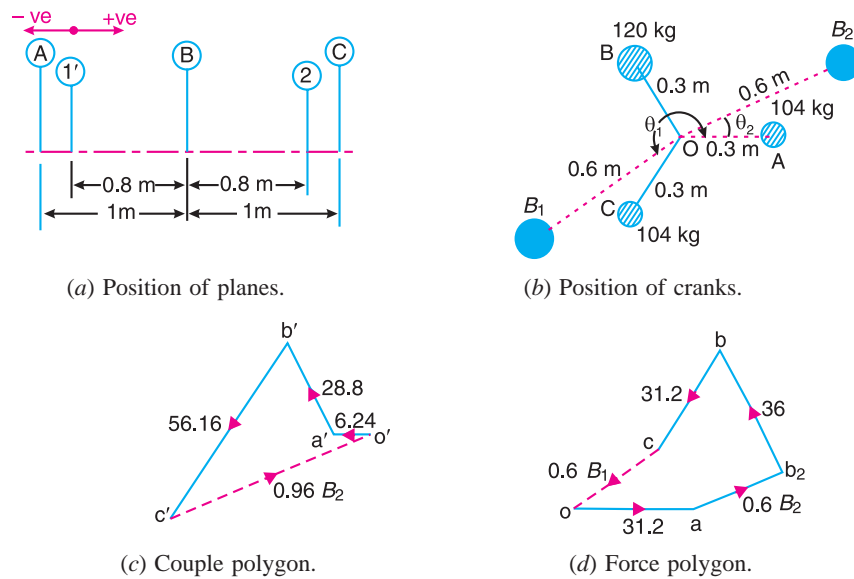


Fig. 22.8

2. Hammer blow per wheel

We know that hammer blow per wheel

$$= B_1 \cdot \omega^2 \cdot b_1 = 57.5 (37.7)^2 20.6 = 49\ 035 \text{ N Ans.}$$



This chamber is used to test the acoustics of a vehicle so that the noise it produces can be reduced. The panels in the walls and ceiling of the room absorb the sound which is monitored (above)

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 22.4. The following data refer to two cylinder locomotive with cranks at 90° :

Reciprocating mass per cylinder = 300 kg ; Crank radius = 0.3 m ; Driving wheel diameter = 1.8 m ; Distance between cylinder centre lines = 0.65 m ; Distance between the driving wheel central planes = 1.55 m.

Determine : **1.** the fraction of the reciprocating masses to be balanced, if the hammer blow is not to exceed 46 kN at 96.5 km. p.h. ; **2.** the variation in tractive effort ; and **3.** the maximum swaying couple.

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Solution. Given : $m = 300 \text{ kg}$; $r = 0.3 \text{ m}$; $D = 1.8 \text{ m}$ or $R = 0.9 \text{ m}$; $a = 0.65 \text{ m}$; Hammer blow = $46 \text{ kN} = 46 \times 10^3 \text{ N}$; $v = 96.5 \text{ km/h} = 26.8 \text{ m/s}$

1. Fraction of the reciprocating masses to be balanced

Let c = Fraction of the reciprocating masses to be balanced, and
 B = Magnitude of balancing mass placed at each of the driving wheels at radius b .

We know that the mass of the reciprocating parts to be balanced
 $= c.m = 300c \text{ kg}$

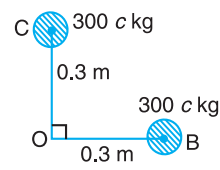
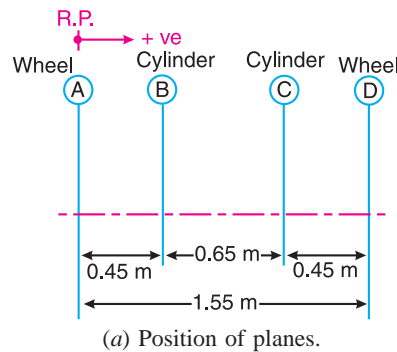


Fig. 22.9

The position of planes of the wheels and cylinders is shown in Fig. 22.9 (a), and the position of cranks is shown in Fig 22.9 (b). Assuming the plane of wheel A as the reference plane, the data may be tabulated as below :

Table 22.3

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from plane A (l)m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
A (R.P.)	B	b	B.b	0	0
B	300 c	0.3	90 c	0.45	40.5 c
C	300 c	0.3	90 c	1.1	99 c
D	B	b	B.b	1.55	1.55 B.b

Now the couple polygon, to some suitable scale, may be drawn with the data given in Table 22.3 (column 6), as shown in Fig. 22.10. The closing side of the polygon (vector $c'o'$) represents the balancing couple and is proportional to $1.55 B.b$.

From the couple polygon,

$$1.55 B.b = \sqrt{(40.5c)^2 + (99c)^2} = 107c$$

$$\therefore B.b = 107 c / 1.55 = 69 c$$

We know that angular speed,

$$\omega = v/R = 26.8/0.9 = 29.8 \text{ rad/s}$$

\therefore Hammer blow,

$$46 \times 10^3 = B. \omega^2 .b$$

$$= 69 c (29.8)^2 = 61\,275 c$$

$$\therefore c = 46 \times 10^3 / 61\,275 = 0.751 \text{ Ans.}$$

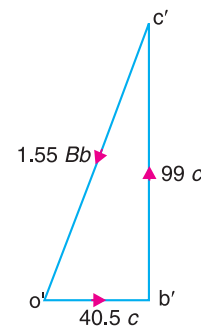


Fig. 22.10

2. Variation in tractive effort

We know that variation in tractive effort

$$= \pm\sqrt{2}(1-c)m\omega^2.r = \pm\sqrt{2}(1-0.751) 300(29.8)^2 0.3$$

$$= 28\,140 \text{ N} = 28.14 \text{ kN Ans.}$$

Maximum swaying couple

We know the maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m\omega^2.r = \frac{0.65(1-0.751)}{\sqrt{2}} \times 300(29.8)^2 0.3 = 9148 \text{ N-m}$$

$$= 9.148 \text{ kN-m Ans.}$$

Example 22.5. The following data apply to an outside cylinder uncoupled locomotive :

Mass of rotating parts per cylinder = 360 kg ; Mass of reciprocating parts per cylinder = 300 kg ; Angle between cranks = 90° ; Crank radius = 0.3 m ; Cylinder centres = 1.75 m ; Radius of balance masses = 0.75 m ; Wheel centres = 1.45 m.

If whole of the rotating and two-thirds of reciprocating parts are to be balanced in planes of the driving wheels, find :

1. Magnitude and angular positions of balance masses,
2. Speed in kilometres per hour at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and
3. Swaying couple at speed arrived at in (2) above.

Solution : Given : $m_1 = 360 \text{ kg}$; $m_2 = 300 \text{ kg}$; $\angle AOD = 90^\circ$; $r_A = r_D = 0.3 \text{ m}$; $a = 1.75 \text{ m}$; $r_B = r_C = 0.75 \text{ m}$; $c = 2/3$.

We know that the equivalent mass of the rotating parts to be balanced per cylinder,

$$m = m_A = m_D = m_1 + c.m_2 = 360 + \frac{2}{3} \times 300 = 560 \text{ kg}$$

1. Magnitude and angular position of balance masses

Let m_B and m_C = Magnitude of the balance masses, and

θ_B and θ_C = angular position of the balance masses m_B and m_C from the crank A.

The magnitude and direction of the balance masses may be determined, graphically, as discussed below :

1. First of all, draw the positions of the planes of the wheels and the cylinders as shown in Fig. 22.11 (a). Since the cranks of the two cylinders are at right angles, therefore assuming the position of the cylinder A in the horizontal direction, draw OA and OD at right angles to each other as shown in Fig. 22.11 (b).
2. Assuming the plane of wheel B as the reference plane, the data may be tabulated as below:

Table 22.4

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from plane B(l) m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
A	560	0.3	168	- 0.15	- 25.2
B (R.P)	m_B	0.75	$0.75 m_B$	0	0
C	m_C	0.75	$0.75 m_C$	1.45	$1.08 m_C$
D	560	0.3	168	1.6	268.8

3. Now draw the couple polygon with the data given in Table 22.4 column (6), to some suitable scale as shown in Fig. 22.11(c). The closing side $d'o'$ represents the balancing couple and it is proportional to $1.08 m_C$. Therefore, by measurement,

$$1.08 m_C = 269.6 \text{ kg-m}^2 \quad \text{or} \quad m_C = 249 \text{ kg} \quad \text{Ans.}$$

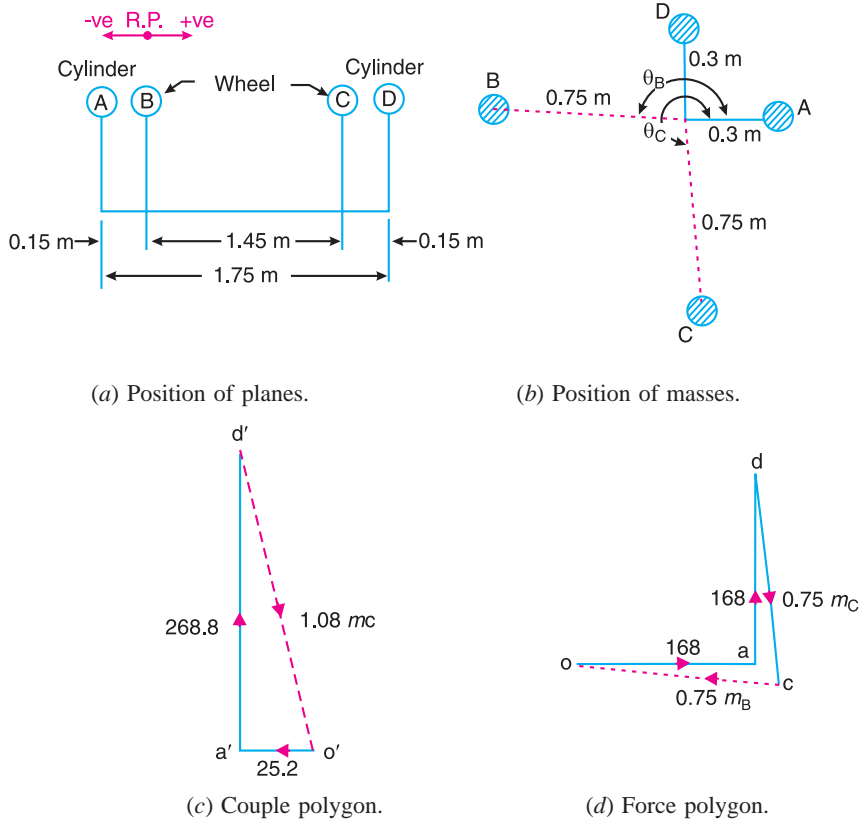


Fig. 22.11

4. To determine the angular position of the balancing mass C, draw OC parallel to vector $d'o'$ as shown in Fig. 22.11 (b). By measurement,

$$\theta_C = 275^\circ \quad \text{Ans.}$$

5. In order to find the balancing mass B, draw the force polygon with the data given in Table 22.4 column (4), to some suitable scale, as shown in Fig. 22.11 (d). The vector co represents

the balancing force and it is proportional to $0.75 m_B$. Therefore, by measurement,

$$0.75 m_B = 186.75 \text{ kg-m} \quad \text{or} \quad m_B = 249 \text{ kg} \quad \text{Ans.}$$

6. To determine the angular position of the balancing mass B , draw OB parallel to vector oc as shown Fig. 22.11 (b). By measurement,

$$\theta_B = 174.5^\circ \quad \text{Ans.}$$

2. Speed at which the wheel will lift off the rails

Given : $P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$; $D = 1.8 \text{ m}$

Let $\omega =$ Angular speed at which the wheels will lift off the rails in rad/s, and

$v =$ Corresponding linear speed in km/h.

We know that each balancing mass,

$$m_B = m_C = 249 \text{ kg}$$

\therefore Balancing mass for reciprocating parts,

$$B = \frac{c.m_2}{m} \times 249 = \frac{2}{3} \times \frac{300}{560} \times 249 = 89 \text{ kg}$$

We know that $\omega = \sqrt{\frac{P}{B.b}} = \sqrt{\frac{30 \times 10^3}{89 \times 0.75}} = 21.2 \text{ rad/s}$...($\because b = r_B = r_C$)

and

$$v = \omega \times D / 2 = 21.2 \times 1.8 / 2 = 19.08 \text{ m/s}$$

$$= 19.08 \times 3600 / 1000 = 68.7 \text{ km/h} \quad \text{Ans.}$$

3. Swaying couple at speed $\omega = 21.1 \text{ rad/s}$

We know that the swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2 \cdot \omega^2 \cdot r = \frac{1.75 \left[1 - \frac{2}{3} \right]}{\sqrt{2}} \times 300 (21.2)^2 \cdot 0.3 \text{ N-m}$$

$$= 16\,687 \text{ N-m} = 16.687 \text{ kN-m} \quad \text{Ans.}$$

22.9. Balancing of Coupled Locomotives

The uncoupled locomotives as discussed in the previous article, are obsolete now-a-days. In a coupled locomotive, the driving wheels are connected to the leading and trailing wheels by an outside coupling rod. By such an arrangement, a greater portion of the engine mass is utilised by tractive purposes. In coupled locomotives, the coupling rod cranks are placed diametrically opposite to the adjacent main cranks (*i.e.* driving cranks). The coupling rods together with cranks and pins may be treated as rotating masses



A dynamo converts mechanical energy into electrical energy.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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and completely balanced by masses in the respective wheels. Thus in a coupled engine, the rotating and reciprocating masses must be treated separately and the balanced masses for the two systems are suitably combined in the wheel.

It may be noted that the variation of pressure between the wheel and the rail (*i.e.* hammer blow) may be reduced by equal distribution of balanced mass (B) between the driving, leading and trailing wheels respectively.

Example 22.6. The following particulars relate to a two-cylinder locomotive with two coupled wheels on each side :

Stroke	= 650 mm
Mass of reciprocating parts per cylinder	= 240 kg
Mass of revolving parts per cylinder	= 200 kg
Mass of each coupling rod	= 250 kg
Radius of centre of coupling rod pin	= 250 mm
Distances between cylinders	= 0.6 m
Distance between wheels	= 1.5 m
Distance between coupling rods	= 1.8 m

The main cranks are at right angles and the coupling rod pins are at 180° to their respective main cranks. The balance masses are to be placed in the wheels at a mean radius of 675 mm in order to balance whole of the revolving and $3/4$ th of the reciprocating masses. The balance mass for the reciprocating masses is to be divided equally between the driving wheels and the coupled wheels. Find : **1.** The magnitudes and angular positions of the masses required for the driving and trailing wheels, and **2.** The hammer blow at 120 km/h, if the wheels are 1.8 metre diameter.

Solution. Given : $L_C = L_D = 650$ mm or $r_C = r_D = 325$ mm = 0.325 m ; $m_1 = 240$ kg ; $m_2 = 200$ kg ; $m_3 = 250$ kg ; $r_A = r_F = 250$ mm = 0.25 m ; $CD = 0.6$ m ; $BE = 1.5$ m ; $AF = 1.8$ m ; $r_B = r_E = 675$ mm = 0.675 m ; $c = 3/4$

The position of planes for the driving wheels B and E , cylinders C and D , and coupling rods A and F , are shown in Fig. 22.12 (a).

The angular position of cranks C and D and coupling pins A and F are shown in Fig. 22.12(b).

We know that mass of the reciprocating parts per cylinder to be balanced

$$= c.m_1 = \frac{3}{4} \times 240 = 180 \text{ kg}$$

Since the reciprocating masses are to be divided equally between the driving wheels and trailing wheels, therefore 90 kg is taken for driving wheels and 90 kg for trailing wheels. Now for each driving wheel, the following masses are to be balanced :

1. Half of the mass of coupling rod *i.e.* $\frac{1}{2} \times 250 = 125$ kg . In other words, the masses at the coupling rods A and F to be balanced for each driving wheel are

$$m_A = m_F = 125 \text{ kg}$$

2. Whole of the revolving mass *i.e.* 200 kg and the mass of the reciprocating parts *i.e.* 90 kg. In other words, total mass at the cylinders C and D to be balanced for each driving wheel are

$$m_C = m_D = 200 + 90 = 290 \text{ kg}$$

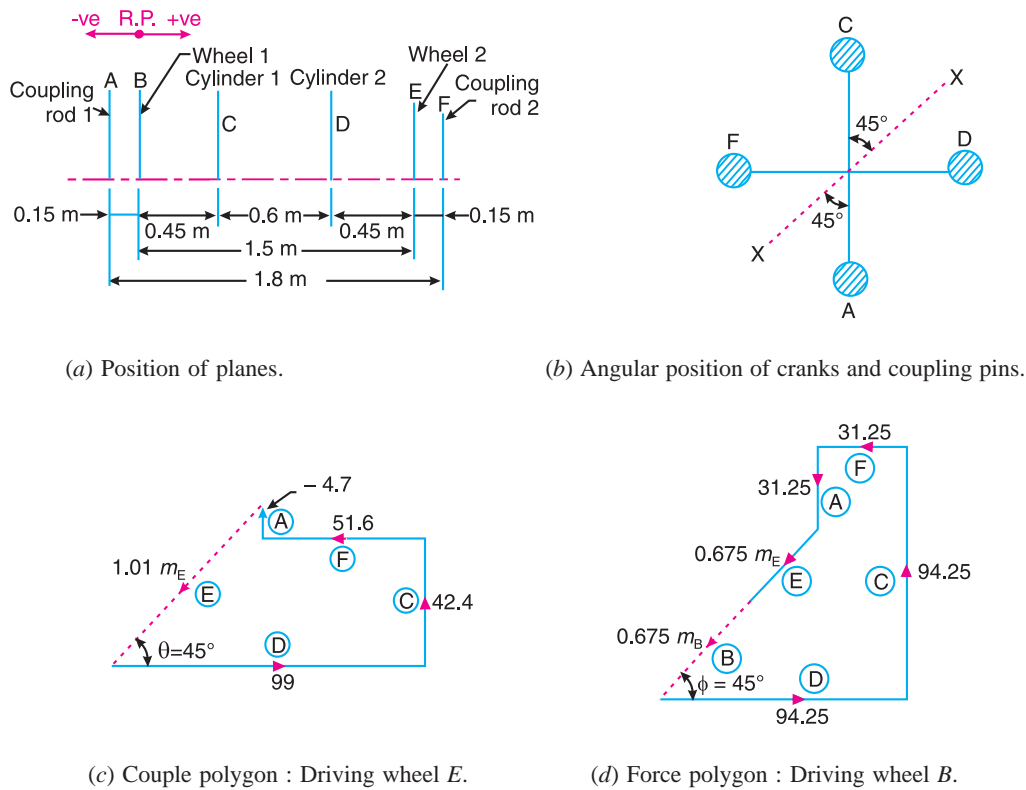


Fig. 22.12

Balanced masses in the driving wheels

Let m_B and m_E be the balance masses placed in the driving wheels B and E respectively. Taking the plane of B as reference plane, the data may be tabulated as below :

Table 22.5. (For driving wheels)

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from Plane B(l) m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
A	125	0.25	31.25	- 0.15	- 4.7
B (R.P.)	m_B	0.675	$0.675 m_B$	0	0
C	290	0.325	94.25	0.45	42.4
D	290	0.325	94.25	1.05	99
E	m_E	0.675	$0.675 m_E$	1.5	$1.01 m_E$
F	125	0.25	31.25	1.65	51.6

In order to find the balance mass m_E in the driving wheel E, draw a couple polygon from the data given in Table 22.5 (column 6), to some suitable scale as shown in Fig 22.12 (c). The closing side of polygon as shown dotted is proportional to $1.01 m_E$. Therefore by measurement, we find that

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$$1.01 m_E = 67.4 \text{ kg-m}^2 \text{ or } m_E = 66.7 \text{ kg Ans.}$$

and $\theta = 45^\circ$ **Ans.**

Now draw the force polygon from the data given in Table 22.5 (column 4), to some suitable scale, as shown in Fig. 22.12 (d). The closing side of the polygon as shown dotted is proportional to $0.675 m_B$. Therefore by measurement, we find that

$$0.675 m_B = 45 \text{ kg-m or } m_B = 66.7 \text{ kg Ans.}$$

and $\phi = 45^\circ$ **Ans.**

Balance masses in the trailing wheels

For each trailing wheel, the following masses are to be balanced :

1. Half of the mass of the coupling rod *i.e.* 125 kg. In other words, the masses at the coupling rods A and F to be balanced for each trailing wheel are

$$m_A = m_F = 125 \text{ kg}$$

2. Mass of the reciprocating parts *i.e.* 90 kg. In other words, the mass at the cylinders C and D to be balanced for each trailing wheel are

$$m_C = m_D = 90 \text{ kg}$$

Let m'_B and m'_E be the balanced masses placed in the trailing wheels. Taking the plane of B as the reference plane, the data may be tabulated as below :

Table 22.6. (For trailing wheels)

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from plane B (l) m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
A	125	0.25	31.25	- 0.15	- 4.7
B (R.P.)	m'_B	0.675	$0.675 m'_B$	0	0
C	90	0.325	29.25	0.45	13.2
D	90	0.325	29.25	1.05	30.7
E	m'_E	0.675	$0.675 m'_E$	1.5	$1.01 m'_E$
F	125	0.25	31.25	1.65	51.6

In order to find the balance mass m'_E in the trailing wheel E, draw a couple polygon from the data given in Table 22.6 (column 6), to some suitable scale, as shown in Fig. 22.13 (a). The closing side of the polygon as shown dotted is proportional to $1.01 m'_E$. Therefore by measurement, we find that

$$1.01 m'_E = 27.5 \text{ m}^2 \text{ or } m'_E = 27.5 \text{ kg Ans.}$$

and $\alpha = 40^\circ$ **Ans.**

Now draw the force polygon from the data given in Table 22.6 (column 4), to some suitable scale, as shown in Fig. 22.13 (b). The closing side of the polygon as shown dotted is proportional to $0.675 m'_B$. Therefore by measurement, we find that

$$0.675 m'_B = 18.35 \text{ kg-m or } m'_B = 27.2 \text{ kg Ans.}$$

and $\beta = 50^\circ$ **Ans.**

Fig. 22.14 shows the balance masses in the four wheels and it will be seen that the balance masses for the driving wheels are symmetrical about the axis X-X [Fig. 22.12 (b)]. Similarly the balance masses for the trailing wheels are symmetrical about the axis X-X.

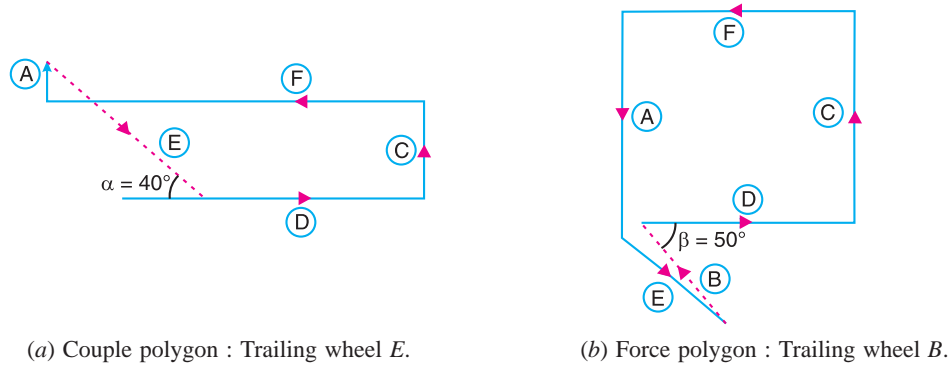


Fig. 22.13

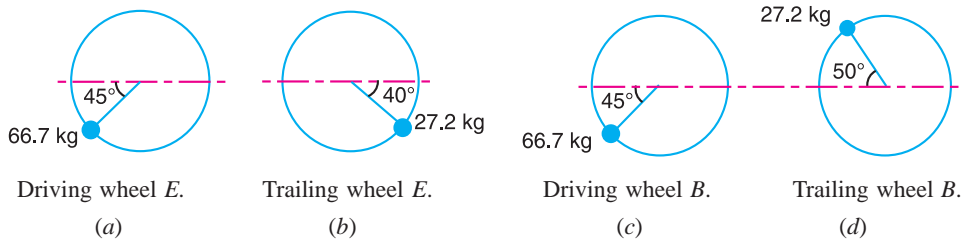


Fig. 22.14

Hammer blow

In order to find the hammer blow, we must find the balance mass required for reciprocating masses only. For this, the data may be tabulated as below. Let m''_B and m''_E be the balanced masses required for the reciprocating masses.

Table 22.7. (For hammer blow)

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from Plane B(l) m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
B(R.P.)	m''_B	0.675	$0.675 m''_B$	0	0
C	90	0.325	29.25	0.45	13.2
D	90	0.325	29.25	1.05	30.7
E	m''_E	0.675	$0.675 m''_E$	1.5	$1.01 m''_E$

Now the couple polygon and the force polygon may be drawn, but due to symmetry we shall only draw the couple polygon from the data given in Table 22.7 (column 6), to some suitable scale as shown in Fig 22.15.

From Fig. 22.15,

$$1.01 m''_E = \sqrt{(30.7)^2 + (13.2)^2} = 33.4$$

$$\therefore m''_E = 33 \text{ kg}$$

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We know that linear speed of the wheel,
 $v = 120 \text{ km/h} = 33.33 \text{ m/s}$
 and diameter of the wheel, $D = 1.8 \text{ m}$

∴ Angular speed of the wheel

$$\omega = \frac{v}{D/2} = \frac{33.33}{1.8/2} = 37 \text{ rad/s}$$

We know that hammer blow

$$= \pm B \cdot \omega^2 \cdot b = 33(37)^2 0.675 = \pm 30.494 \text{ N Ans.}$$

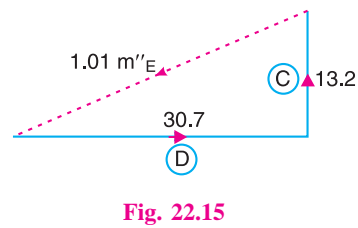


Fig. 22.15

... (∵ $B = m''_E$, and $b = r_B = r_E$)

22.10. Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as **In-line engines**. The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine :

1. The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must *close ; and
2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close.

We have already discussed, that the primary unbalanced force due to the reciprocating masses is equal to the component, parallel to the line of stroke, of the centrifugal force produced by the equal mass placed at the crankpin and revolving with it. Therefore, in order to give the **primary balance of the reciprocating parts of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses.**

Notes : 1. For a two cylinder engine with cranks at 180° , condition (1) may be satisfied, but this will result in an unbalanced couple. Thus the above method of primary balancing cannot be applied in this case.

2. For a three cylinder engine with cranks at 120° and if the reciprocating masses per cylinder are same, then condition (1) will be satisfied because the forces may be represented by the sides of an equilateral triangle. However, by taking a reference plane through one of the cylinder centre lines, two couples with non-parallel axes will remain and these cannot vanish vectorially. Hence the above method of balancing fails in this case also.



The speedometer is an instrument which shows how fast a car is moving. It works with a magnet that spins around as the car moves.

Note : This picture is given as additional information and is not a direct example of the current chapter.

* The closing side of the primary force polygon gives the maximum unbalanced primary force and the closing side of the primary couple polygon gives the maximum unbalanced primary couple.

3. For a four cylinder engine, similar reasoning will show that complete primary balance is possible and it follows that

‘For a multi-cylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided that the number of cranks are not less than four’.

22.11. Balancing of Secondary Forces of Multi-cylinder In-line Engines

When the connecting rod is not too long (*i.e.* when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.

We have discussed in Art. 22.2, that the secondary force,

$$F_s = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

This expression may be written as

$$F_s = m.(2\omega)^2 \times \frac{r}{4n} \times \cos 2\theta$$

As in case of primary forces, the secondary forces may be considered to be equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed at the imaginary crank of length $r/4n$ and revolving at twice the speed of the actual crank (*i.e.* 2ω) as shown in Fig. 22.16.

Thus, in multi-cylinder in-line engines, each imaginary secondary crank with a mass attached to the crankpin is inclined to the line of stroke at twice the angle of the actual crank. The values of the secondary forces and couples may be obtained by considering the revolving mass. This is done in the similar way as discussed for primary forces. The following two conditions must be satisfied in order to give a complete secondary balance of an engine :

1. The algebraic sum of the secondary forces must be equal to zero. In other words, the secondary force polygon must close, and
2. The algebraic sum of the couples about any point in the plane of the secondary forces must be equal to zero. In other words, the secondary couple polygon must close.

Note : The closing side of the secondary force polygon gives the maximum unbalanced secondary force and the closing side of the secondary couple polygon gives the maximum unbalanced secondary couple.

Example 22.7. A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.

Solution. Given $r_1 = r_2 = r_3 = r_4 = 150 \text{ mm} = 0.15 \text{ m}$; $m_1 = 50 \text{ kg}$; $m_2 = 60 \text{ kg}$; $m_4 = 50 \text{ kg}$

We have discussed in Art. 22.10 that in order to give the primary balance of the reciprocating parts of a multi-cylinder engine, the problem may be treated as that of revolving masses with the reciprocating masses transferred to their respective crank pins.

The position of planes is shown in Fig. 22.17 (a). Assuming the plane of third cylinder as the reference plane, the data may be tabulated as given in Table 22.8.

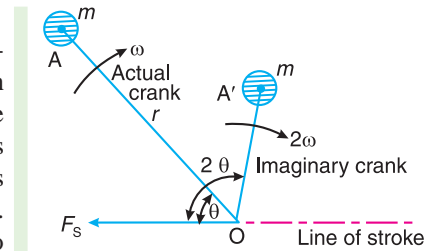


Fig. 22.16. Secondary force.

Table 22.8

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from plane 3(l) m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
1	50	0.15	7.5	- 0.4	- 3
2	60	0.15	9	- 0.2	- 1.8
3(R.P.)	m_3	0.15	$0.15m_3$	0	0
4	50	0.15	7.5	0.2	1.5

First of all, the angular position of cranks 2 and 4 are obtained by drawing the couple polygon from the data given in Table 22.8 (column 6). Assume the position of crank 1 in the horizontal direction as shown in Fig 22.17 (b), The couple polygon, as shown in Fig. 22.17 (c), is drawn as discussed below:

1. Draw vector $o'a'$ in the horizontal direction (i.e. parallel to $O1$) and equal to $- 3 \text{ kg-m}^2$, to some suitable scale.
2. From point o' and a' , draw vectors $o'b'$ and $a'b'$ equal to $- 1.8 \text{ kg-m}^2$ and 1.5 kg-m^2 respectively. These vectors intersect at b' .

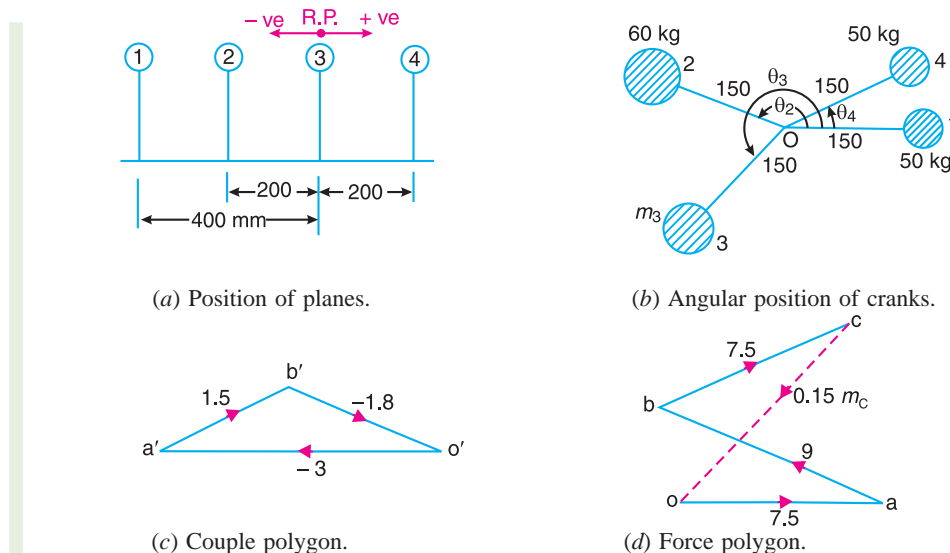


Fig. 22.17

3. Now in Fig. 22.17 (b), draw $O2$ parallel to vector $o'b'$ and $O4$ parallel to vector $a'b'$.

By measurement, we find that the angular position of crank 2 from crank 1 in the anticlockwise direction is

$$\theta_2 = 160^\circ \text{ Ans.}$$

and the angular position of crank 4 from crank 1 in the anticlockwise direction is

$$\theta_4 = 26^\circ \text{ Ans.}$$

In order to find the mass of the third cylinder (m_3) and its angular position, draw the force polygon, to some suitable scale, as shown in Fig. 22.17 (d), from the data given in Table 22.8 (column 4). Since the closing side of the force polygon (vector co) is proportional to $0.15 m_3$, therefore by measurement,

$$0.15m_3 = 9 \text{ kg-m} \quad \text{or} \quad m_3 = 60 \text{ kg Ans.}$$

Now draw $O3$ in Fig 22.17 (b), parallel to vector co . By measurement, we find that the angular position of crank 3 from crank 1 in the anticlockwise direction is

$$\theta_3 = 227^\circ \text{ Ans.}$$

Example 22.8. A four crank engine has the two outer cranks set at 120° to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks.

If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force ?

Solution. Given : $m_1 = m_4 = 400 \text{ kg}$; $r = 300 \text{ mm} = 0.3 \text{ m}$; $l = 1.2 \text{ m}$; $N = 240 \text{ r.p.m.}$ or $\omega = 2\pi \times 240 / 60 = 25.14 \text{ rad/s}$

Reciprocating mass and the relative angular position for each of the inner cranks

Let m_2 and $m_3 =$ Reciprocating mass for the inner cranks 2 and 3 respectively, and θ_2 and $\theta_3 =$ Angular positions of the cranks 2 and 3 with respect to crank 1 respectively.

The position of the planes of rotation of the cranks and their angular setting are shown in Fig. 22.18 (a) and (b) respectively. Taking the plane of crank 2 as the reference plane, the data may be tabulated as below :

Table 22.9

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from plane (2) (l) m (5)	Couple $\div \omega^2$ (m.r.l.) kg-m ² (6)
1	400	0.3	120	- 0.45	- 54
2(R.P.)	m_2	0.3	$0.3 m_2$	0	0
3	m_3	0.3	$0.3 m_3$	0.75	$0.225 m_3$
4	400	0.3	120	1.35	162

Since the engine is to be in complete primary balance, therefore the primary couple polygon and the primary force polygon must close. First of all, the primary couple polygon, as shown in Fig. 22.18 (c), is drawn to some suitable scale from the data given in Table 22.9 (column 6), in order to find the reciprocating mass for crank 3. Now by measurement, we find that

$$0.225 m_3 = 196 \text{ kg-m}^2 \quad \text{or} \quad m_3 = 871 \text{ kg Ans.}$$

and its angular position with respect to crank 1 in the anticlockwise direction,

$$\theta_3 = 326^\circ \text{ Ans.}$$

Now in order to find the reciprocating mass for crank 2, draw the primary force polygon, as shown in Fig. 22.18 (d), to some suitable scale from the data given in Table 22.9 (column 4). Now by measurement, we find that

$$0.3 m_2 = 284 \text{ kg-m} \quad \text{or} \quad m_2 = 947 \text{ kg Ans.}$$

and its angular position with respect to crank 1 in the anticlockwise direction,

$$\theta_2 = 168^\circ \text{ Ans.}$$

Maximum secondary unbalanced force

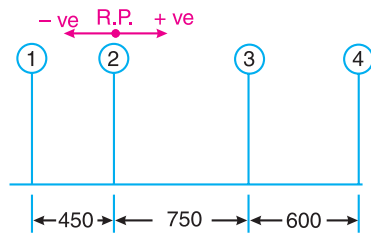
The secondary crank positions obtained by rotating the primary cranks at twice the angle,

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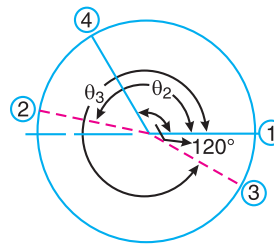
is shown in Fig. 22.18 (e). Now draw the secondary force polygon, as shown in Fig. 22.18 (f), to some suitable scale, from the data given in Table 22.9 (column 4). The closing side of the polygon shown dotted in Fig. 22.18 (f) represents the maximum secondary unbalanced force. By measurement, we find that the maximum secondary unbalanced force is proportional to 582 kg-m.

∴ Maximum secondary unbalanced force

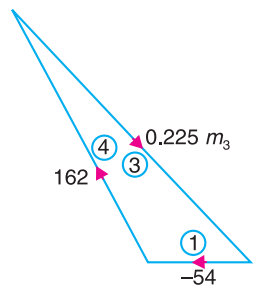
$$= 582 \times \frac{\omega^2}{n} = \frac{582(25.14)^2}{1.2/0.3} = 91\,960\text{N} = 91.96\text{ kN Ans.} \quad \dots (\because n = l/r)$$



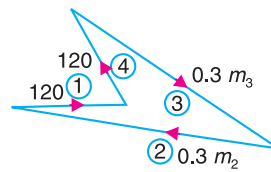
(a) Positions of planes.



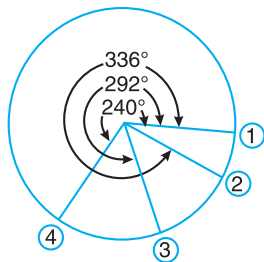
(b) Primary crank positions.



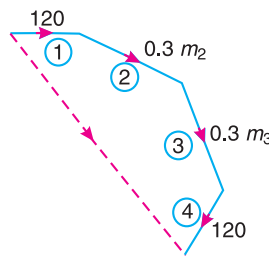
(c) Primary couple polygon.



(d) Primary force polygon.



(e) Secondary crank positions.



(f) Secondary force polygon.

Fig. 22.18

Example 22.9. The cranks and connecting rods of a 4-cylinder in-line engine running at 1800 r.p.m. are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg.

Determine : 1. Unbalanced primary and secondary forces, if any, and 2. Unbalanced primary and secondary couples with reference to central plane of the engine.

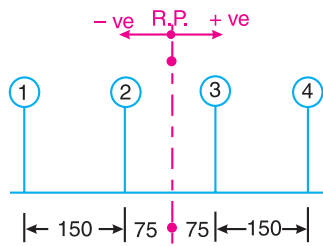
Solution. Given : $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.52$ rad/s ; $r = 60$ mm = 0.06 m ; $l = 240$ mm = 0.24 m ; $m = 1.5$ kg

1. Unbalanced primary and secondary forces

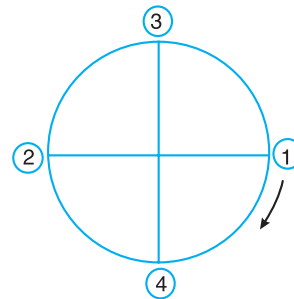
The position of the cylinder planes and cranks is shown in Fig.22.19 (a) and (b) respectively. With reference to central plane of the engine, the data may be tabulated as below :

Table 22.10

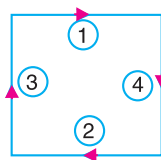
Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from ref. plane 3 (l) m (5)	Couple $\div \omega^2$ (m.r.l.) kg-m ² (6)
1	1.5	0.6	0.9	- 0.225	- 0.2025
2	1.5	0.6	0.9	- 0.075	- 0.0675
3	1.5	0.6	0.9	+ 0.075	+ 0.0675
4	1.5	0.6	0.9	+ 0.225	+ 0.2025



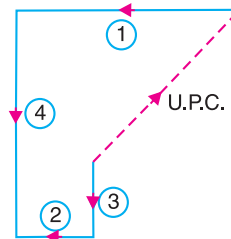
(a) Cylinder plane positions.



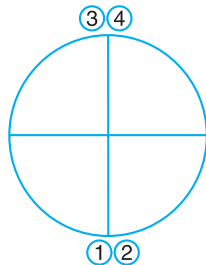
(b) Primary crank positions.



(c) Primary force polygon.



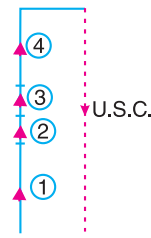
(d) Primary couple polygon.



(e) Secondary crank positions.



(f) Secondary force polygon.



(g) Secondary couple polygon.

Fig. 22.19

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The primary force polygon from the data given in Table 22.10 (column 4) is drawn as shown in Fig. 22.19 (c). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces. **Ans.**

The secondary crank positions, taking crank 3 as the reference crank, is shown in Fig. 22.19 (e). From the secondary force polygon as shown in Fig. 22.19 (f), we see that it is a closed figure. Therefore there are no unbalanced secondary forces. **Ans.**

2. Unbalanced primary and secondary couples

The primary couple polygon from the data given in Table 22.10 (column 6) is drawn as shown in Fig. 22.19 (d). The closing side of the polygon, shown dotted in the figure, represents unbalanced primary couple. By measurement, we find the unbalanced primary couple is proportional to 0.19 kg-m^2 .

∴ Unbalanced primary couple,

$$U.P.C = 0.19 \times \omega^2 = 0.19 (188.52)^2 = 6752 \text{ N-m Ans.}$$

The secondary couple polygon is shown in Fig. 22.1 (g). The unbalanced secondary couple is shown by dotted line. By measurement, we find that unbalanced secondary couple is proportional to 0.54 kg-m^2 .

∴ Unbalanced secondary couple,

$$U.S.C. = 0.54 \times \frac{\omega^2}{n} = 0.54 \times \frac{(188.52)^2}{0.24/0.6} = 4798 \text{ N-m Ans.} \quad \dots (\because n = l/r)$$

Example 22.10. Fig. 22.20 shows the arrangement of the cranks in a four crank symmetrical engine in which the masses of the reciprocating parts at cranks 1 and 4 are each equal to m_1 and at cranks 2 and 3 are each equal to m_2 .

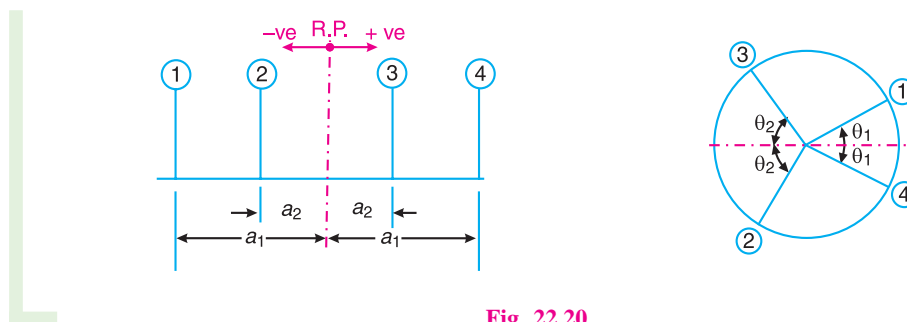


Fig. 22.20

Show that the arrangement is balanced for primary forces and couples and for secondary forces provided that

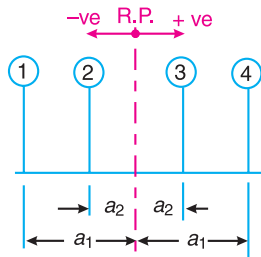
$$\frac{m_1}{m_2} = \frac{\cos \theta_2}{\cos \theta_1} ; \frac{a_1}{a_2} = \frac{\tan \theta_2}{\tan \theta_1}, \quad \text{and} \quad \cos \theta_1 \cdot \cos \theta_2 = \frac{1}{2}.$$

Solution. Given : Mass of reciprocating parts at cranks 1 and 4 = m_1 ; Mass of the reciprocating parts at cranks 2 and 3 = m_2

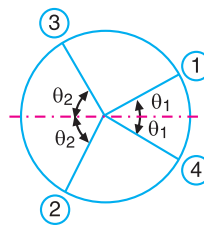
The position of planes and primary and secondary crank positions are shown in Fig. 22.21 (a), (b) and (c) respectively. Assuming the reference plane midway between the planes of rotation of cranks 2 and 3, the data may be tabulated as below :

Table 22.11

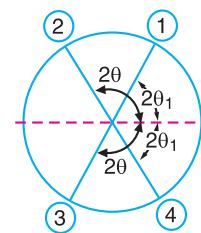
Plane (1)	Mass (m) (2)	Radius (r) (3)	Cent. force ÷ ω ² (m.r) (4)	Distance from ref. plane (l) (5)	Couple ÷ ω ² (m.r.l) (6)
1	m ₁	r	m ₁ .r	- a ₁	- m ₁ .r.a ₁
2	m ₂	r	m ₂ .r	- a ₂	- m ₂ .r.a ₂
3	m ₂	r	m ₂ .r	+ a ₂	+ m ₂ .r.a ₂
4	m ₁	r	m ₁ .r	+ a ₁	+ m ₁ .r.a ₁



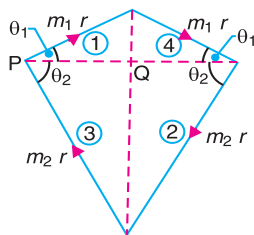
(a) Position of planes.



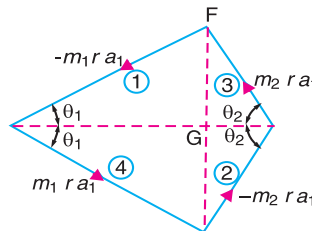
(b) Primary crank positions.



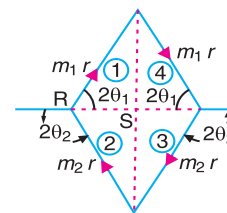
(c) Secondary crank positions.



(d) Primary force polygon.



(e) Primary couple polygon.



(f) Secondary force polygon.

Fig. 22.21

In order to balance the arrangement for primary forces and couples, the primary force and couple polygons must close. Fig. 22.21 (d) and (e) show the primary force and couple polygons, which are closed figures. From Fig. 22.21 (d),

$$PQ = m_1 \cdot r \cos \theta_1 = m_2 \cdot r \cos \theta_2 \quad \text{or} \quad \frac{m_1}{m_2} = \frac{\cos \theta_2}{\cos \theta_1} \quad \text{Ans.}$$

From Fig. 22.21 (e),

$$FG = m_1 \cdot r \cdot a_1 \sin \theta_1 = m_2 \cdot r \cdot a_2 \sin \theta_2$$

or $m_1 \cdot a_1 \sin \theta_1 = m_2 \cdot a_2 \sin \theta_2$

$$\frac{m_1}{m_2} \times \frac{a_1}{a_2} = \frac{\sin \theta_2}{\sin \theta_1} \quad \text{or} \quad \frac{\cos \theta_2}{\cos \theta_1} \times \frac{a_1}{a_2} = \frac{\sin \theta_2}{\sin \theta_1} \quad \dots \left(\because \frac{m_1}{m_2} = \frac{\cos \theta_2}{\cos \theta_1} \right)$$

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$$\therefore \frac{a_1}{a_2} = \frac{\sin \theta_2}{\sin \theta_1} \times \frac{\cos \theta_1}{\cos \theta_2} = \frac{\tan \theta_2}{\tan \theta_1} \quad \text{Ans.}$$

In order to balance the arrangement for secondary forces, the secondary force polygon must close. The position of the secondary cranks is shown in Fig. 22.21 (c) and the secondary force polygon is shown in Fig. 22.21 (f).

Now from Fig. 22.21 (f),

$$RS = m_1 \cdot r \cos 2\theta_1 = m_2 \cdot r \cos(180^\circ - 2\theta_2)$$

or $m_1 \cdot \cos 2\theta_2 = -m_2 \cdot \cos 2\theta_2$

$$\therefore \frac{m_1}{m_2} = \frac{-\cos 2\theta_2}{\cos 2\theta_1} = \frac{-(2\cos^2 \theta_2 - 1)}{2\cos^2 \theta_1 - 1} \dots (\because \cos 2\theta = 2\cos^2 \theta - 1)$$

$$\frac{\cos \theta_2}{\cos \theta_1} = \frac{(1 - 2\cos^2 \theta_2)}{2\cos^2 \theta_1 - 1} \dots \left[\because \frac{m_1}{m_2} = \frac{\cos \theta_2}{\cos \theta_1} \right]$$

$$2\cos^2 \theta_1 \cdot \cos \theta_2 - \cos \theta_2 = \cos \theta_1 - 2\cos^2 \theta_2 \cdot \cos \theta_1$$

$$2\cos \theta_1 \cdot \cos \theta_2 (\cos \theta_1 + \cos \theta_2) = \cos \theta_1 + \cos \theta_2$$

$$2\cos \theta_1 \cdot \cos \theta_2 = 1 \quad \text{or} \quad \cos \theta_1 \cdot \cos \theta_2 = \frac{1}{2} \quad \text{Ans.}$$

Example 22.11. A four cylinder engine has cranks arranged symmetrically along the shaft as shown in Fig. 22.22. The distance between the outer cranks A and D is 5.4 metres and that between the inner cranks B and C is 2.4 metres. The mass of the reciprocating parts belonging to each of the inner cylinders is 2 tonnes, and that belonging to each of the inner cylinders is m tonnes.

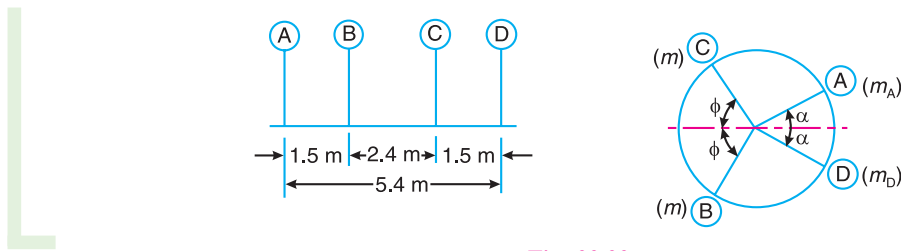


Fig. 22.22

If the primary and secondary forces are to be balanced and also the primary couples, determine the crank angle positions and the mass of the reciprocating parts (m) corresponding to the inner cylinders.

Find also the maximum value of the unbalanced secondary couple, if the stroke is 1 metre, the connecting rod length 2 metres, and the speed of the engine is 110 r.p.m.

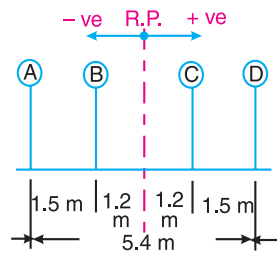
Solution. Given : $AD = 5.4 \text{ m}$; $BC = 2.4 \text{ m}$; $m_A = m_D = 2 \text{ t}$; $L = 1 \text{ m}$ or $r = L/2 = 0.5 \text{ m}$; $l = 2 \text{ m}$; $N = 110 \text{ r.p.m.}$ or $\omega = 2\pi \times 110/60 = 11.52 \text{ rad/s}$

Fig. 22.23 (a) shows the position of planes and Fig. 22.23 (b) shows the end view of the cranks with primary crank angles α and ϕ which are to be determined. Assuming the reference

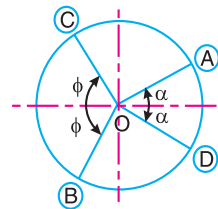
plane mid-way between the planes of rotation of cranks A and D, the data may be tabulated as below :

Table 22.12

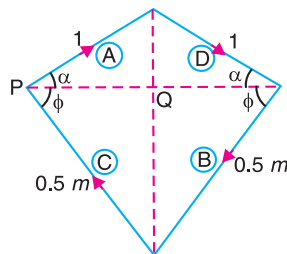
Plane (1)	Mass (m) t (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) t-m (3)	Distance from ref. plane (l) m (4)	Couple $\div \omega^2$ (m.r.l) t-m ² (5)
A	2	0.5	1	- 2.7	- 2.7
B	m	0.5	0.5 m	- 1.2	- 0.6 m
C	m	0.5	0.5 m	+ 1.2	+ 0.6 m
D	2	0.5	1	+ 2.7	+ 2.7



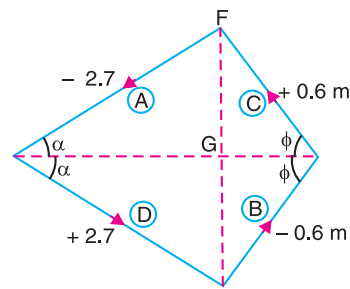
(a) Positions of planes.



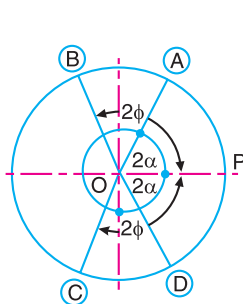
(b) Primary crank positions.



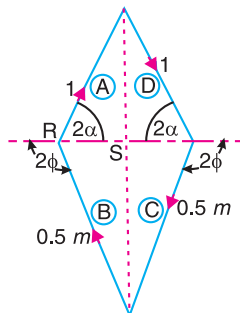
(c) Primary force polygon.



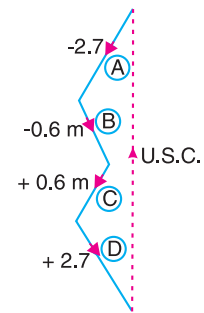
(d) Primary couple polygon.



(e) Secondary crank positions.



(f) Secondary force polygon.



(g) Secondary couple polygon.

Fig. 22.23

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Since the primary forces and couples are to be balanced, therefore the primary force and couple polygons, drawn from the data given in Table 22.12 column (4) and (6) respectively, as shown in Fig. 22.23 (c) and (d), must close.

From Fig. 22.23 (c),

$$PQ = 1 \cos \alpha = 0.5 m \cos \phi$$

$$\therefore \cos \phi = \frac{1 \cos \alpha}{0.5 m} = \frac{2 \cos \alpha}{m} \quad \dots (i)$$



A Steam-powered ship.

From Fig. 22.23 (d),

$$FG = 2.7 \sin \alpha = 0.6 m \sin \phi$$

$$\therefore \sin \alpha = \frac{0.6 m \sin \phi}{2.7} = \frac{m \sin \phi}{4.5} \quad \dots (ii)$$

Now draw the secondary crank positions as shown in Fig. 22.23 (e). Let OP be the reference line. The secondary crank angles are given below :

$$OP \text{ to } OA = 2\alpha$$

$$OP \text{ to } OC = 2(180^\circ - \phi) = 360^\circ - 2\phi$$

$$OP \text{ to } OB = 2(180^\circ + \phi) = 360^\circ + 2\phi$$

$$OP \text{ to } OD = 2(360^\circ - \alpha) = 720^\circ - 2\alpha$$

Since the secondary forces are to be balanced, therefore the secondary force polygon, as shown in Fig. 22.23 (f), must close. Now from Fig. 22.23 (f),

$$RS = 1 \cos 2\alpha = 0.5 m \cos (180^\circ - 2\phi)$$

or
$$\frac{1}{0.5m} = \frac{-\cos 2\phi}{\cos 2\alpha} = \frac{-(2\cos^2 \phi - 1)}{2\cos^2 \alpha - 1} \quad \dots (\because \cos 2\theta = 2\cos^2 \theta - 1)$$

$$2 \cos^2 \alpha - 1 = 0.5m(1 - 2 \cos^2 \phi) = 0.5m \left[1 - 2 \left(\frac{2 \cos \alpha}{m} \right)^2 \right] \quad \dots \text{ [From equation (i)]}$$

$$= 0.5m \left[1 - \frac{8 \cos^2 \alpha}{m^2} \right] = 0.5m - \frac{4 \cos^2 \alpha}{m}$$

$$2 \cos^2 \alpha + \frac{4 \cos^2 \alpha}{m} = 1 + 0.5m \quad \text{or} \quad \cos^2 \alpha \left(\frac{2m + 4}{m} \right) = 1 + 0.5m$$

$$\therefore \cos^2 \alpha = (1 + 0.5m) \times \frac{m}{2m + 4} = \frac{m}{4} \quad \dots \text{ (iii)}$$

Now from equation (ii)

$$\sin^2 \alpha = \left(\frac{m \sin \phi}{4.5} \right)^2$$

$$\text{or} \quad 1 - \cos^2 \alpha = \frac{m^2 \sin^2 \phi}{20.25} = \frac{m^2}{20.25} (1 - \cos^2 \phi) = \frac{m^2}{20.25} \left[1 - \left(\frac{2 \cos \alpha}{m} \right)^2 \right] \quad \dots \text{ [From equations (i)]}$$

$$1 - \frac{m}{4} = \frac{m^2}{20.25} \left(1 - \frac{4}{m^2} \times \frac{m}{4} \right) = \frac{m^2}{20.25} \left(1 - \frac{1}{m} \right) = \frac{m^2}{20.25} - \frac{m}{20.25}$$

\dots [From equation (iii)]

$$\text{or} \quad \frac{m^2}{20.25} - \frac{m}{20.25} + \frac{m}{4} - 1 = 0 \quad \text{or} \quad m^2 + 4.0625m - 20.25 = 0$$

$$\therefore m = \frac{-4.0625 \pm \sqrt{(4.0625)^2 + 4 \times 20.25}}{2} = 2.9 \text{ t}$$

$$\text{We know that } \cos^2 \alpha = \frac{m}{4} = \frac{2.9}{4} = 0.725$$

$$\therefore \cos \alpha = 0.851 \quad \text{or} \quad \alpha = 31.6^\circ \quad \text{Ans.}$$

$$\text{Also} \quad \cos \phi = \frac{2 \cos \alpha}{m} = \frac{2 \times 0.851}{2.9} = 0.5869 \quad \text{or} \quad \phi = 54.06^\circ \quad \text{Ans.}$$

Maximum unbalanced secondary couple

The secondary couple polygon is shown in Fig. 22.23 (g). The maximum unbalanced secondary couple is shown by a dotted line. By measurement, we find that the maximum unbalanced secondary couple is proportional to $8 \text{ t}\cdot\text{m}^2$.

\therefore Maximum unbalanced secondary couple,

$$U.S.C = 8 \times \frac{\omega^2}{n} = 8 \times \frac{(11.52)^2}{2/0.5} = 265.4 \text{ kN}\cdot\text{m} \quad \text{Ans.} \quad \dots (\because n = l/r)$$

Example 22.12. A five cylinder in-line engine running at 750 r.p.m. has successive cranks 144° apart, the distance between the cylinder centre lines being 375 mm. The piston stroke is 225 mm and the ratio of the connecting rod to the crank is 4. Examine the engine for balance of primary and secondary forces and couples. Find the maximum values of these and the position of the central crank at which these maximum values occur. The reciprocating mass for each cylinder is 15 kg.

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Solution. Given : $N = 750$ r.p.m. or $\omega = 2\pi \times 750/60 = 78.55$ rad/s ; $L = 225$ mm = 0.225 m or $r = 0.1125$ m ; $n = l/r = 4$; $m = 15$ kg

Assuming the engine to be a vertical engine, the positions of the cylinders and the cranks are shown in Fig. 22.24 (a), (b) and (c). The plane 3 may be taken as the reference plane and the crank 3 as the reference crank. The data may be tabulated as given in the following table.

Table 22.13

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg-m (4)	Distance from ref. Plane 3 (l) m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
1	15	0.1125	1.6875	- 0.75	- 1.265
2	15	0.1125	1.6875	- 0.375	- 0.6328
3(R.P.)	15	0.1125	1.6875	0	0
4	15	0.1125	1.6875	+ 0.375	+ 0.6328
5	15	0.1125	1.6875	+ 0.75	+ 1.265

Now, draw the force and couple polygons for primary and secondary cranks as shown in Fig. 22.24 (d), (e), (f), and (g). Since the primary and secondary force polygons are close, therefore the engine is balanced for primary and secondary forces. **Ans.**

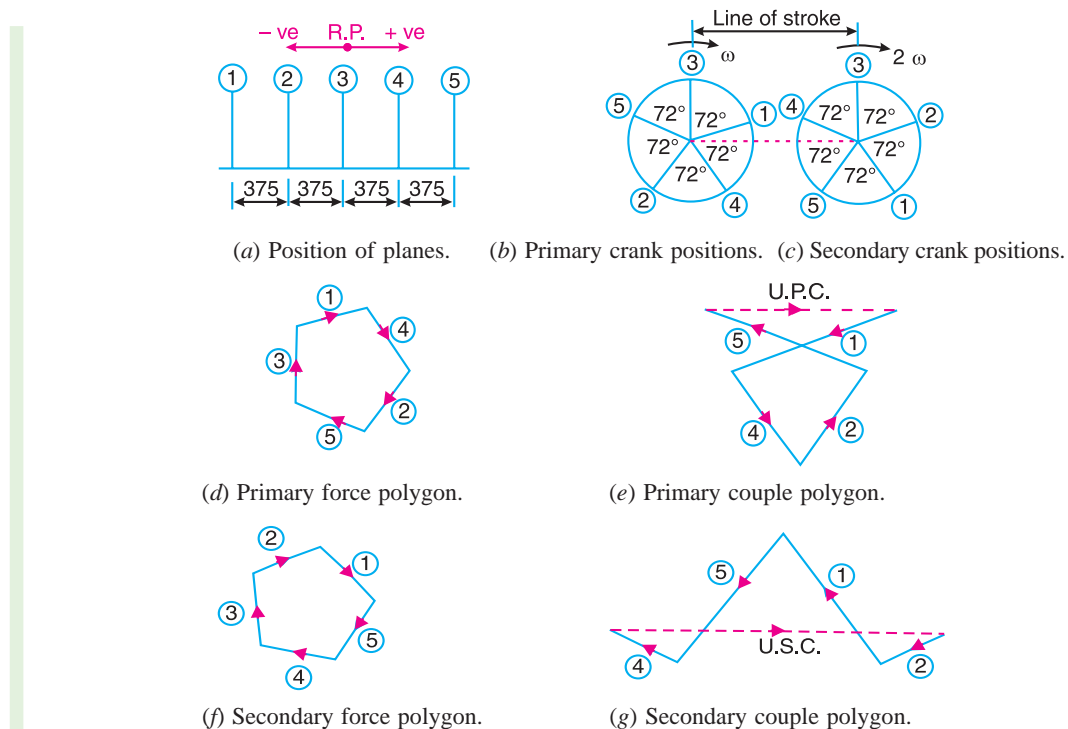


Fig. 22.24

Maximum unbalanced primary couple

We know that the closing side of the primary couple polygon [shown dotted in Fig. 22.24 (e)] gives the maximum unbalanced primary couple. By measurement, we find that maximum unbalanced primary couple is proportional to 1.62 kg-m².

∴ Maximum unbalanced primary couple,

$$U.P.C. = 1.62 \times \omega^2 = 1.62 (78.55)^2 = 9996 \text{ N-m Ans.}$$

We see from Fig. 22.24 (e) [shown by dotted line] that the maximum unbalanced primary couple occurs when crank 3 is at 90° from the line of stroke.

Maximum unbalanced secondary couple

We know that the closing side of the secondary couple polygon [shown dotted in Fig. 22.24 (g)] gives the maximum unbalanced secondary couple. By measurement, we find that maximum unbalanced secondary couple is proportional to 2.7 kg-m².

∴ Maximum unbalanced secondary couple.

$$U.S.C = 2.7 \times \frac{\omega^2}{n} = 2.7 \times \frac{(78.55)^2}{4} = 4165 \text{ N-m Ans.}$$

We see from Fig. 22.24 (g) that if the vector representing the unbalanced secondary couple (shown by dotted line) is rotated through 90°, it will coincide with the line of stroke. Hence the original crank will be rotated through 45°. Therefore, the maximum unbalanced secondary couple occurs when crank 3 is at 45° and at successive intervals of 90° (i.e. 135°, 225° and 315°) from the line of stroke.

Example 22.13. The firing order in a 6 cylinder vertical four stroke in-line engine is 1-4-2-6-3-5. The piston stroke is 100 mm and the length of each connecting rod is 200 mm. The pitch distances between the cylinder centre lines are 100 mm, 100 mm, 150 mm, 100 mm, and 100 mm respectively. The reciprocating mass per cylinder is 1 kg and the engine runs at 3000 r.p.m.

Determine the out-of-balance primary and secondary forces and couples on this engine, taking a plane midway between the cylinder 3 and 4 as the reference plane.

Solution. Given : $L = 100 \text{ mm}$ or $r = L / 2 = 50 \text{ mm} = 0.05 \text{ m}$; $l = 200 \text{ mm}$; $m = 1 \text{ kg}$; $N = 3000 \text{ r.p.m.}$

The position of the cylinders and the cranks are shown in Fig. 22.25 (a), (b) and (c). With the reference plane midway between the cylinders 3 and 4, the data may be tabulated as given in the following table :

Table 22.14

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω ² (m.r) kg-m (4)	Distance from plane 3 (l)m (5)	Couple ÷ ω ² (m.r.l) kg-m ² (6)
1	1	0.05	0.05	- 0.275	- 0.01375
2	1	0.05	0.05	- 0.175	- 0.00875
3	1	0.05	0.05	- 0.075	- 0.00375
4	1	0.05	0.05	+ 0.075	+ 0.00375
5	1	0.05	0.05	+ 0.175	+ 0.00875
6	1	0.05	0.05	+ 0.275	+ 0.01375

Now, draw the force and couple polygons for the primary and secondary cranks as shown in Fig. 22.25 (d), (e), (f) and (g).

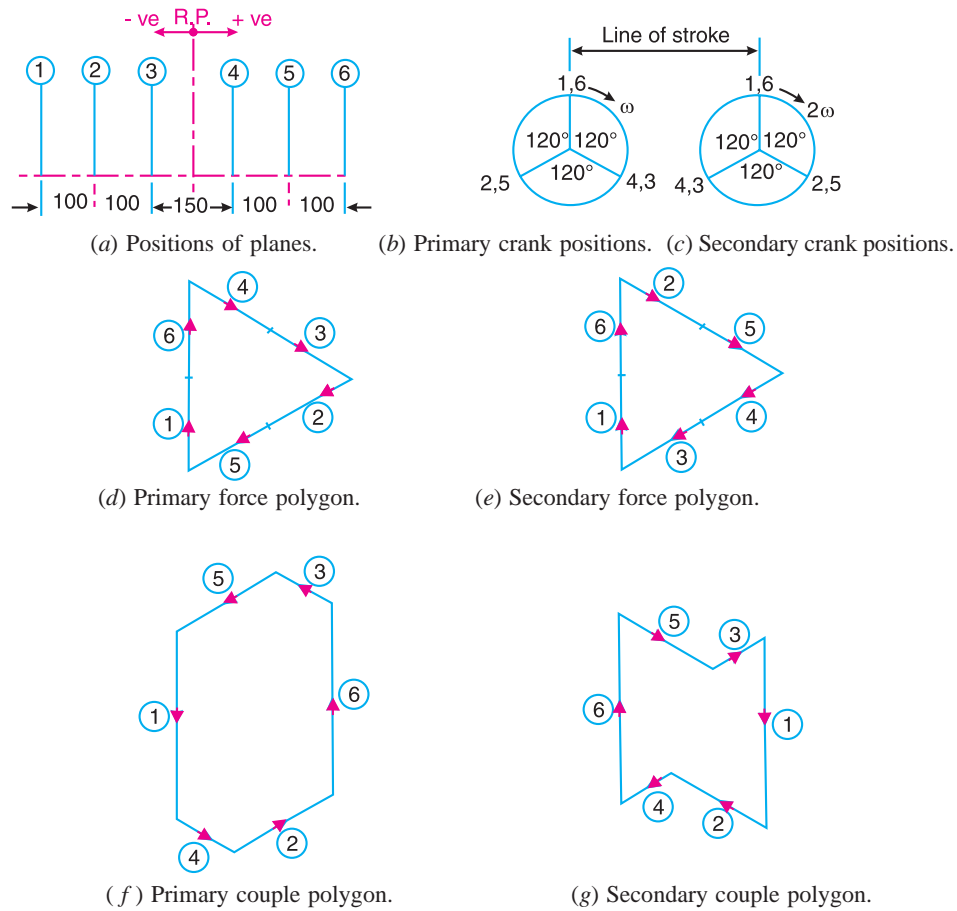


Fig. 22.25

From Fig. 22.25 (d) and (e), we see that the primary and secondary force polygons are closed figures, therefore there are no out-of-balance primary and secondary forces. Thus the engine is balanced for primary and secondary forces. Also, the primary and secondary couple polygons, as shown in Fig. 22.25 (f) and (g) are closed figures, therefore there are no out-of-balance primary and secondary couples. Thus the engine is balanced for primary and secondary couples. **Ans.**

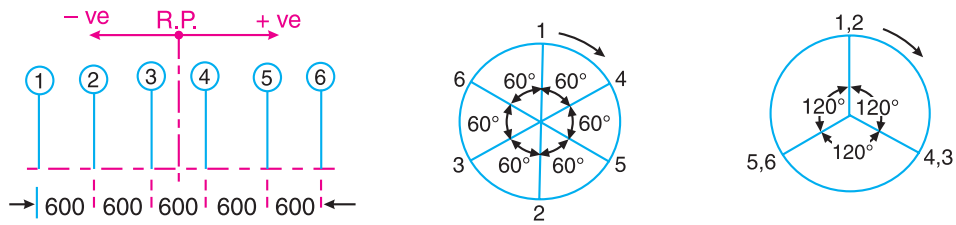
Example 22.14. In an in-line six cylinder engine working on two stroke cycle, the cylinder centre lines are spaced at 600 mm. In the end view, the cranks are 60° apart and in the order 1-4-5-2-3-6. The stroke of each piston is 400 mm and the connecting rod length is 1 metre. The mass of the reciprocating parts is 200 kg per cylinder and that of rotating parts 100 kg per crank. The engine rotates at 300 r.p.m. Examine the engine for the balance of primary and secondary forces and couples. Find the maximum unbalanced forces and couples.

Solution. Given : $L = 400$ mm or $r = L/2 = 200$ mm = 0.2 m ; $l = 1$ m ; $m_1 = 200$ kg ; $m_2 = 100$ kg ; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

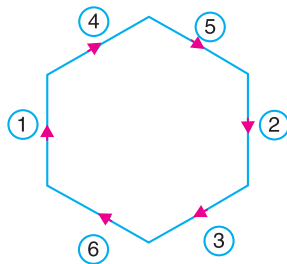
Assuming the engine to be a vertical engine, the position of planes of cylinders and the angular position of primary and secondary cranks (assuming the crank 1 coinciding with the line of stroke *i.e.* in the vertical direction) are shown in Fig. 22.26 (a), (b) and (c) respectively. It may be noted that the mass of rotating parts (m_2) at each crank pin is included with the mass of reciprocating parts (m_1) for primary forces and couples only. Taking the reference plane between the cylinders 3 and 4, the data may be tabulated as below:

Table 22.15. (For primary forces and couples only)

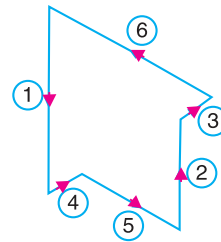
Plane (1)	Mass (m) kg $m = m_1 + m_2$ (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r)kg-m (4)	Distance from ref. plane (1) m (5)	Couple $\div \omega^2$ (m.r.l) kg-m ² (6)
1	300	0.2	60	- 1.5	- 90
2	300	0.2	60	- 0.9	- 54
3	300	0.2	60	- 0.3	- 18
4	300	0.2	60	+ 0.3	+ 18
5	300	0.2	60	+ 0.9	+ 54
6	300	0.2	60	+ 1.5	+ 90



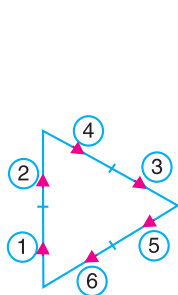
(a) Positions of planes of cylinders. (b) Primary crank positions. (c) Secondary crank positions.



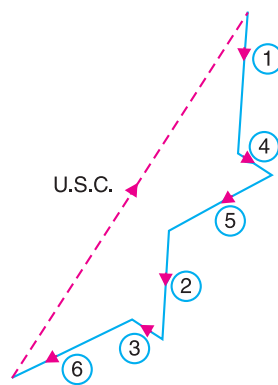
(d) Primary force polygon.



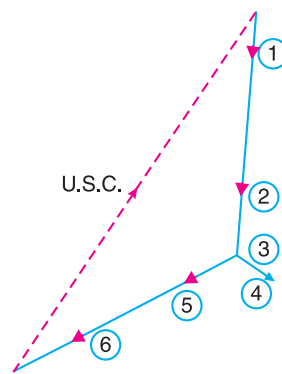
(e) Primary couple polygon.



(f) Secondary force polygon.



(g) Secondary couple polygon.



(h) Secondary couple polygon.

Fig. 22.26

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Now draw the force polygon and couple polygon for primary cranks from the data given in Table 22.15 (column 4 and 6) respectively, as shown in Fig. 22.26 (d) and (e). Since the force and couple polygons are closed figures, therefore the engine is balanced for primary force and couple (*i.e.* there is no unbalanced primary force and couple).

The data for the secondary forces and couples, taking $m = m_1 = 200$ kg, may be tabulated as below :

Table 22.16. (For secondary forces and couples)

Plane	Mass (m) kg $m = m_1$	Radius (r) m	Cent. force $\div \omega^2$ (m.r) kg-m	Distance from ref. plane (l) m	Couple $\div \omega^2$ (m.r.l) kg-m ²
1	200	0.2	40	- 1.5	- 60
2	200	0.2	40	- 0.9	- 36
3	200	0.2	40	- 0.3	- 12
4	200	0.2	40	+ 0.3	+ 12
5	200	0.2	40	+ 0.9	+ 36
6	200	0.2	40	+ 1.5	+ 60

First of all, draw the secondary force polygon for secondary cranks [the angular position of which is shown in Fig. 22.26 (c)] from the data given in Table 22.16 (column 4) as shown in Fig. 22.26 (f). Since the secondary force polygon is a closed figure, therefore the engine is balanced for secondary forces (*i.e.* there is no unbalanced secondary forces.) Now draw the secondary couple polygon for the secondary cranks from the data given in Table 22.16 (column 6) as shown in Fig. 22.26 (g). The closing side of the polygon as shown by dotted line represents the maximum unbalanced secondary couple. By measurement, we find that maximum unbalanced couple is proportional to 168 kg-m^2 .

∴ Maximum unbalanced secondary couple

$$= 168 \times \frac{\omega^2}{n} = 168 \times \frac{(31.42)^2}{1/0.2} = 33\ 170 \text{ N-m} = 33.17 \text{ kN-m Ans.}$$

... (∵ $n = l/r$)

Note : The secondary couple polygon may also be drawn as shown in Fig. 22.26 (h).

22.12. Balancing of Radial Engines (Direct and Reverse Cranks Method)

The method of direct and reverse cranks is used in balancing of radial or V-engines, in which the connecting rods are connected to a common crank. Since the plane of rotation of the various cranks (in radial or V-engines) is same, therefore there is no unbalanced primary or secondary couple.

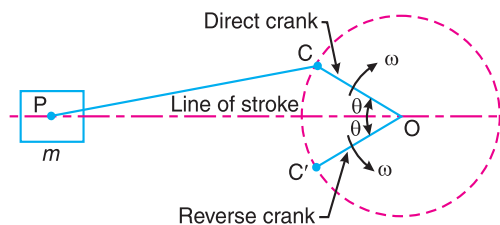


Fig. 22.27. Reciprocating engine mechanism.

Consider a reciprocating engine mechanism as shown in Fig. 22.27. Let the crank OC (known as the direct crank) rotates uniformly at ω radians per second in a clockwise direction. Let at any instant the crank makes an angle θ with the line of stroke OP . The indirect or reverse crank OC' is the image of the direct crank OC , when seen through the mirror placed at the line of stroke. A little consideration will show that when the direct crank revolves in a clockwise direction, the reverse crank will revolve in the anticlockwise direction. We shall now discuss the primary and secondary forces due to the mass (m) of the reciprocating parts at P .

Considering the primary forces

We have already discussed that primary force is $m\omega^2.r \cos \theta$. This force is equal to the component of the centrifugal force along the line of stroke, produced by a mass (m) placed at the crank pin C . Now let us suppose that the mass (m) of the reciprocating parts is divided into two parts, each equal to $m/2$.

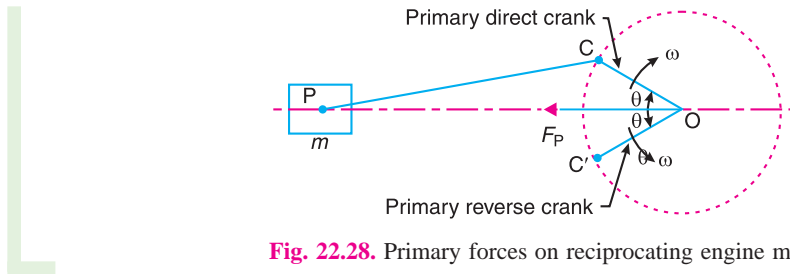


Fig. 22.28. Primary forces on reciprocating engine mechanism.

It is assumed that $m/2$ is fixed at the **direct crank** (termed as **primary direct crank**) pin C and $m/2$ at the **reverse crank** (termed as **primary reverse crank**) pin C' , as shown in Fig. 22.28. We know that the centrifugal force acting on the primary direct and reverse crank

$$= \frac{m}{2} \times \omega^2 . r$$

∴ Component of the centrifugal force acting on the primary direct crank

$$= \frac{m}{2} \times \omega^2 . r \cos \theta \quad \dots \text{(in the direction from } O \text{ to } P)$$

and, the component of the centrifugal force acting on the primary reverse crank

$$= \frac{m}{2} \times \omega^2 . r \cos \theta \quad \dots \text{(in the direction from } O \text{ to } P)$$

∴ Total component of the centrifugal force along the line of stroke

$$= 2 \times \frac{m}{2} \times \omega^2 . r \cos \theta = m\omega^2 . r \cos \theta = \text{Primary force, } F_p$$

Hence, for primary effects the mass m of the reciprocating parts at P may be replaced by two masses at C and C' each of magnitude $m/2$.

Note : The component of the centrifugal forces of the direct and reverse cranks, in a direction perpendicular to the line of stroke, are each equal to $\frac{m}{2} \times \omega^2 . r \sin \theta$, but opposite in direction. Hence these components are balanced.

Considering secondary forces

We know that the secondary force

$$= m(2\omega)^2 \frac{r}{4n} \times \cos 2\theta = m.\omega^2 r. \times \frac{\cos 2\theta}{n}$$



A diesel train engine.

In the similar way as discussed above, it will be seen that for the secondary effects, the mass (m) of the reciprocating parts may be replaced by two masses (each $m/2$) placed at D and D' such that $OD = OD' = r/4n$. The crank OD is the *secondary direct crank* and rotates at 2ω rad/s in the clockwise direction, while the crank OD' is the *secondary reverse crank* and rotates at 2ω rad/s in the anticlockwise direction as shown in Fig. 22.29.

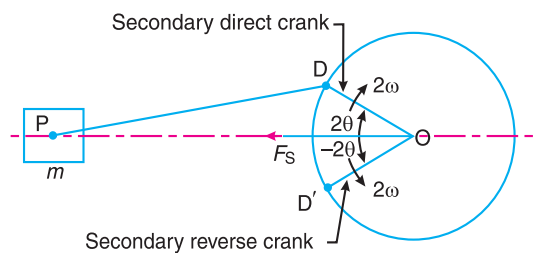


Fig. 22.29. Secondary force on reciprocating engine mechanism.

Example 22.15. The three cylinders of an air compressor have their axes 120° to one another, and their connecting rods are coupled to a single crank. The stroke is 100 mm and the length of each connecting rod is 150 mm. The mass of the reciprocating parts per cylinder is 1.5 kg. Find the maximum primary and secondary forces acting on the frame of the compressor when running at 3000 r.p.m. Describe clearly a method by which such forces may be balanced.

Solution. Given : $L = 100 \text{ mm}$ or $r = L/2 = 50 \text{ mm} = 0.05 \text{ m}$; $l = 150 \text{ mm} = 0.15 \text{ m}$;
 $m = 1.5 \text{ kg}$; $N = 3000 \text{ r.p.m.}$ or $\omega = 2\pi \times 3000/60 = 314.2 \text{ rad/s}$

The position of three cylinders is shown in Fig. 22.30. Let the common crank be along the inner dead centre of cylinder 1. Since common crank rotates clockwise, therefore θ is positive when measured clockwise.

Maximum primary force acting on the frame of the compressor

The primary direct and reverse crank positions as shown in Fig. 22.31 (a) and (b), are obtained as discussed below :

1. Since $\theta = 0^\circ$ for cylinder 1, therefore both the primary direct and reverse cranks will coincide with the common crank.
2. Since $\theta = \pm 120^\circ$ for cylinder 2, therefore the primary direct crank is 120° clockwise and the primary reverse crank is 120° anti-clockwise from the line of stroke of cylinder 2.
3. Since $\theta = \pm 240^\circ$ for cylinder 3, therefore the primary direct crank is 240° clockwise and the primary reverse crank is 240° anti-clockwise from the line of stroke of cylinder 3.

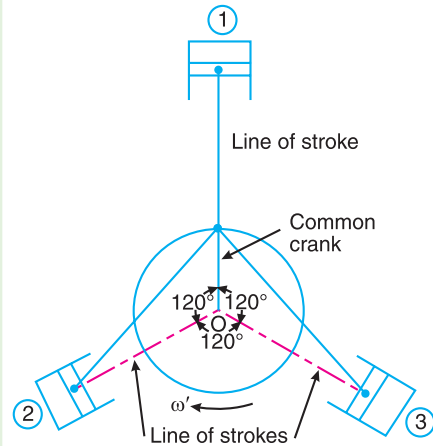


Fig. 22.30

From Fig. 22.31 (b), we see that the primary reverse cranks form a balanced system. Therefore there is no unbalanced primary force due to the reverse cranks. From Fig. 22.31 (a), we see that the resultant primary force is equivalent to the centrifugal force of a mass $3m/2$ attached to the end of the crank.

$$\therefore \text{Maximum primary force} = \frac{3m}{2} \times \omega^2 \cdot r = \frac{3 \times 1.5}{2} (314.2)^2 \cdot 0.05 = 11\,106 \text{ N} = 11.106 \text{ kN Ans.}$$

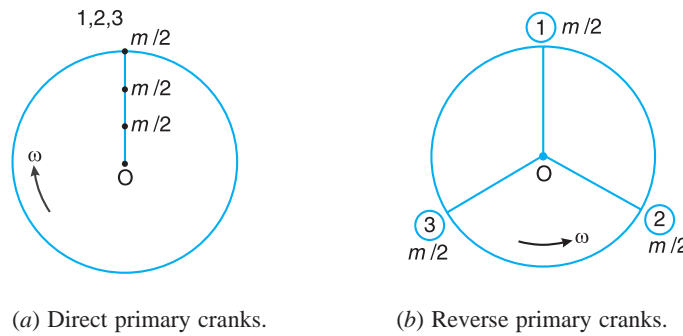


Fig. 22.31

The maximum primary force may be balanced by a mass attached diametrically opposite to the crank pin and rotating with the crank, of magnitude B_1 at radius b_1 such that

$$B_1 \cdot b_1 = \frac{3m}{2} \times r = \frac{3 \times 1.5}{2} \times 0.05 = 0.1125 \text{ N-m Ans.}$$

Maximum secondary force acting on the frame of the compressor

The secondary direct and reverse crank positions as shown in Fig. 22.32 (a) and (b), are obtained as discussed below :

1. Since $\theta = 0^\circ$ and $2\theta = 0^\circ$ for cylinder 1, therefore both the secondary direct and reverse cranks will coincide with the common crank.
2. Since $\theta = \pm 120^\circ$ and $2\theta = \pm 240^\circ$ for cylinder 2, therefore the secondary direct crank is 240° clockwise and the secondary reverse crank is 240° anticlockwise from the line of stroke of cylinder 2.
3. Since $\theta = \pm 240^\circ$ and $2\theta = \pm 480^\circ$, therefore the secondary direct crank is 480° or 120° clockwise and the secondary reverse crank is 480° or 120° anti-clockwise from the line of stroke of cylinder 3.

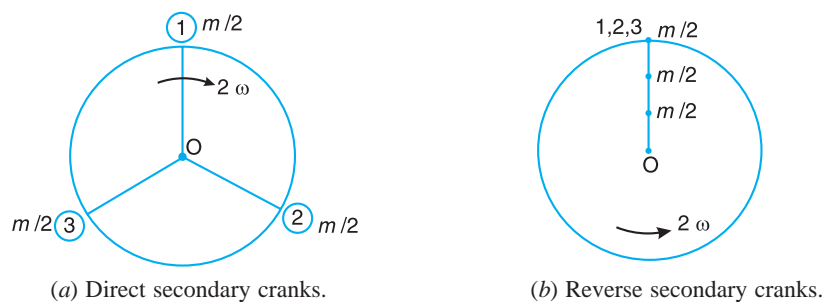


Fig. 22.32

From Fig. 22.32 (a), we see that the secondary direct cranks form a balanced system. Therefore there is no unbalanced secondary force due to the direct cranks. From Fig. 22.32 (b), we see that the resultant secondary force is equivalent to the centrifugal force of a mass $3 m/2$ attached at a crank radius of $r/4n$ and rotating at a speed of 2ω rad/s in the opposite direction to the crank.



Submarines are powered by diesel or nuclear powered engines which have reciprocating and rotating parts.

∴ Maximum secondary force

$$= \frac{2m}{2} (2\omega)^2 \left(\frac{r}{4n} \right) = \frac{3 \times 1.5}{2} (2 \times 314.2)^2 \left[\frac{0.05}{4 \times 0.15 / 0.05} \right] \text{N}$$

... (∵ $n = l/r$)

$$= 3702 \text{ N Ans.}$$

This maximum secondary force may be balanced by a mass B_2 at radius b_2 , attached diametrically opposite to the crankpin, and rotating anti-clockwise at twice the crank speed, such that

$$B_2 \cdot b_2 = \frac{3m}{2} \times \frac{r}{4n} = \frac{3 \times 1.5}{2} \times \frac{0.05}{4 \times 0.15 / 0.05} = 0.009 \text{ 375 N-m Ans.}$$

Notes : 1. Proceeding in the same way as discussed in the above example, we may prove that in a radial engine with an odd number of cylinders, the primary forces may be balanced by attaching single mass of magnitude $\frac{1}{2} K.m$ (K being the number of cylinders), at crank radius diametrically opposite to the crank pin.

2. For a radial engine containing four or more cylinders, the secondary direct and reverse cranks form a balanced system, *i.e.* the secondary forces are in complete balance.

22.13. Balancing of V-engines

Consider a symmetrical two cylinder V-engine as shown in Fig. 22.33, The common crank OC is driven by two connecting rods PC and QC . The lines of stroke OP and OQ are inclined to the vertical OY , at an angle α as shown in Fig 22.33.

- Let
- m = Mass of reciprocating parts per cylinder,
 - l = Length of connecting rod,
 - r = Radius of crank,
 - n = Ratio of length of connecting rod to crank radius = l / r
 - θ = Inclination of crank to the vertical at any instant,
 - ω = Angular velocity of crank.

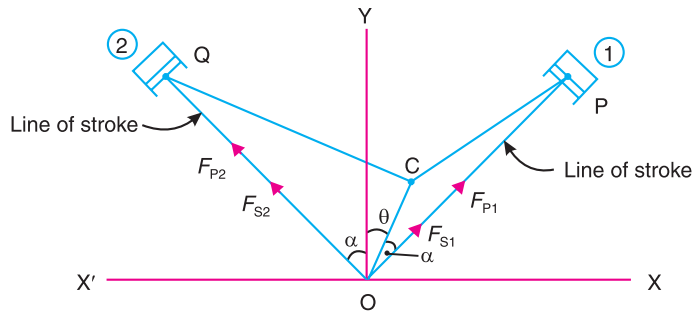


Fig.22.33. Balancing of V-engines.

We know that inertia force due to reciprocating parts of cylinder 1, along the line of stroke

$$= m \cdot \omega^2 \cdot r \left[\cos(\alpha - \theta) + \frac{\cos 2(\alpha - \theta)}{n} \right]$$

and the inertia force due to reciprocating parts of cylinder 2, along the line of stroke

$$= m \cdot \omega^2 \cdot r \left[\cos(\alpha - \theta) + \frac{\cos 2(\alpha + \theta)}{n} \right]$$

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The balancing of V-engines is only considered for primary and secondary forces* as discussed below :

Considering primary forces

We know that primary force acting along the line of stroke of cylinder 1,

$$F_{P1} = m.\omega^2.r \cos(\alpha - \theta)$$

∴ Component of F_{P1} along the vertical line OY ,

$$= F_{P1} \cos \alpha = m.\omega^2.r \cos(\alpha - \theta) \cos \alpha \quad \dots (i)$$

and component of F_{P1} along the horizontal line OX

$$= F_{P1} \sin \alpha = m.\omega^2.r \cos(\alpha - \theta) \sin \alpha \quad \dots (ii)$$

Similarly, primary force acting along the line of stroke of cylinder 2,

$$F_{P2} = m.\omega^2.r \cos(\alpha + \theta)$$

∴ Component of F_{P2} along the vertical line OY

$$= F_{P2} \cos \alpha = m.\omega^2.r \cos(\alpha + \theta) \cos \alpha \quad \dots (iii)$$

and component of F_{P2} along the horizontal line OX'

$$= F_{P2} \sin \alpha = m.\omega^2.r \cos(\alpha + \theta) \sin \alpha \quad \dots (iv)$$

Total component of primary force along the vertical line OY

$$\begin{aligned} F_{PV} &= (i) + (iii) = m.\omega^2.r \cos \alpha [\cos(\alpha - \theta) + \cos(\alpha + \theta)] \\ &= m.\omega^2.r \cos \alpha \times 2 \cos \alpha \cos \theta \\ &\dots [\because \cos(\alpha - \theta) + \cos(\alpha + \theta) = 2 \cos \alpha \cos \theta] \\ &= 2 m.\omega^2.r \cos^2 \alpha \cos \theta \end{aligned}$$

and total component of primary force along the horizontal line OX

$$\begin{aligned} F_{PH} &= (ii) - (iv) = m.\omega^2.r \sin \alpha [\cos(\alpha - \theta) - \cos(\alpha + \theta)] \\ &= m.\omega^2.r \sin \alpha \times 2 \sin \alpha \sin \theta \\ &\dots [\because \cos(\alpha - \theta) - \cos(\alpha + \theta) = 2 \sin \alpha \sin \theta] \\ &= 2 m.\omega^2.r \sin^2 \alpha \sin \theta \end{aligned}$$

∴ Resultant primary force,

$$\begin{aligned} F_P &= \sqrt{(F_{PV})^2 + (F_{PH})^2} \\ &= 2 m.\omega^2.r \sqrt{(\cos^2 \alpha \cos \theta)^2 + (\sin^2 \alpha \sin \theta)^2} \quad \dots (v) \end{aligned}$$

Notes : The following results, derived from equation (v), depending upon the value of α may be noted :

1. When $2\alpha = 60^\circ$ or $\alpha = 30^\circ$,

$$F_P = 2 m.\omega^2.r \sqrt{(\cos^2 30^\circ \cos \theta)^2 + (\sin^2 30^\circ \sin \theta)^2}$$

* Since the plane of rotation of the crank is same, therefore there are no unbalanced primary and secondary couples.

$$= 2m.\omega^2.r.\sqrt{\left(\frac{3}{4}\cos\theta\right)^2 + \left(\frac{1}{4}\sin\theta\right)^2} = \frac{m}{2}\times\omega^2.r\sqrt{9\cos^2\theta + \sin^2\theta} \quad \dots(vi)$$

2. When $2\alpha = 90^\circ$ or $\alpha = 45^\circ$

$$\begin{aligned} F_P &= 2m.\omega^2.r.\sqrt{(\cos^2 45^\circ \cos\theta)^2 + (\sin^2 45^\circ \sin\theta)^2} \\ &= 2m.\omega^2.r.\sqrt{\left(\frac{1}{2}\cos\theta\right)^2 + \left(\frac{1}{2}\sin\theta\right)^2} = m.\omega^2.r \quad \dots (vii) \end{aligned}$$

3. When $2\alpha = 120^\circ$ or $\alpha = 60^\circ$,

$$\begin{aligned} F_P &= 2m.\omega^2.r.\sqrt{(\cos^2 60^\circ \cos\theta)^2 + (\sin^2 60^\circ \sin\theta)^2} \\ &= 2m.\omega^2.r.\sqrt{\left(\frac{1}{4}\cos\theta\right)^2 + \left(\frac{3}{4}\sin\theta\right)^2} = \frac{m}{2}\times\omega^2.r\sqrt{\cos^2\theta + 9\sin^2\theta} \quad \dots (viii) \end{aligned}$$

Considering secondary forces

We know that secondary force acting along the line of stroke of cylinder 1,

$$F_{S1} = m.\omega^2.r \times \frac{\cos 2(\alpha - \theta)}{n}$$

∴ Component of F_{S1} along the vertical line OY

$$= F_{S1} \cos \alpha = m.\omega^2.r \times \frac{\cos 2(\alpha - \theta)}{n} \times \cos \alpha \quad \dots (ix)$$

and component of F_{S1} along the horizontal line OX

$$= F_{S1} \sin \alpha = m.\omega^2.r \times \frac{\cos 2(\alpha - \theta)}{n} \times \sin \alpha \quad \dots (x)$$

Similarly, secondary force acting along the line of stroke of cylinder 2,

$$F_{S2} = m.\omega^2.r \times \frac{\cos 2(\alpha + \theta)}{n}$$

∴ Component of F_{S2} along the vertical line OY

$$= F_{S2} \cos \alpha = m.\omega^2.r \times \frac{\cos 2(\alpha + \theta)}{n} \times \cos \alpha \quad \dots (xi)$$

and component of F_{S2} along the horizontal line OX'

$$= F_{S2} \sin \alpha = m.\omega^2.r \times \frac{\cos 2(\alpha + \theta)}{n} \times \sin \alpha \quad \dots (xii)$$

Total component of secondary force along the vertical line OY ,

$$\begin{aligned} F_{SV} &= (ix) + (xi) = \frac{m}{n} \times \omega^2.r \cos \alpha [\cos 2(\alpha - \theta) + \cos 2(\alpha + \theta)] \\ &= \frac{m}{n} \times \omega^2.r \cos \alpha \times 2 \cos 2\alpha \cos 2\theta = \frac{2m}{n} \times \omega^2.r \cos \alpha \cdot \cos 2\alpha \cos 2\theta \end{aligned}$$

and total component of secondary force along the horizontal line OX ,

$$\begin{aligned} F_{SH} &= (x) - (xii) = \frac{m}{n} \times \omega^2.r \sin \alpha [\cos 2(\alpha - \theta) - \cos 2(\alpha + \theta)] \\ &= \frac{m}{n} \times \omega^2.r \sin \alpha \times 2 \sin 2\alpha \cdot \sin 2\theta \\ &= \frac{2m}{n} \times \omega^2.r \sin \alpha \cdot \sin 2\alpha \cdot \sin 2\theta \end{aligned}$$

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∴ Resultant secondary force,

$$F_S = \sqrt{(F_{SV})^2 + (F_{SH})^2}$$

$$= \frac{2m}{n} \times \omega^2 \cdot r \sqrt{(\cos \alpha \cdot \cos 2\alpha \cdot \cos 2\theta)^2 + (\sin \alpha \cdot \sin 2\alpha \cdot \sin 2\theta)^2}$$

... (xiii)

Notes : The following results, derived from equation (xiii), depending upon the value of α , may be noted.

1. When $2\alpha = 60^\circ$ or $\alpha = 30^\circ$,

$$F_S = \frac{2m}{n} \times \omega^2 \cdot r \sqrt{(\cos 30^\circ \cos 60^\circ \cos 2\theta)^2 + (\sin 30^\circ \sin 60^\circ \sin 2\theta)^2}$$

$$= \frac{2m}{n} \times \omega^2 \cdot r \sqrt{\left[\frac{\sqrt{3}}{2} \times \frac{1}{2} \cos 2\theta \right]^2 + \left[\frac{1}{2} \times \frac{\sqrt{3}}{2} \sin 2\theta \right]^2}$$

$$= \frac{\sqrt{3}}{2} \times \frac{m}{n} \times \omega^2 \cdot r \quad \dots (xiv)$$

2. When $2\alpha = 90^\circ$ or $\alpha = 45^\circ$,

$$F_S = \frac{2m}{n} \times \omega^2 \cdot r \sqrt{(\cos 45^\circ \cos 90^\circ \cos 2\theta)^2 + (\sin 45^\circ \sin 90^\circ \sin 2\theta)^2}$$

$$= \frac{2m}{n} \times \omega^2 \cdot r \sqrt{0 + \left[\frac{1}{\sqrt{2}} \times 1 \times \sin 2\theta \right]^2} = \frac{\sqrt{2}m}{n} \times \omega^2 \cdot r \sin 2\theta \quad \dots (xv)$$



Automated Guided Vehicles, AGVs, operate in many factories. They ferry goods and materials along carefully marked routes. Many AGVs are guided by signals from electrical loops buried under factory floors.

Note : This picture is given as additional information and is not a direct example of the current chapter.

3. When $2\alpha = 120^\circ$ or $\alpha = 60^\circ$

$$\begin{aligned} F_S &= \frac{2m}{n} \times \omega^2 \cdot r \sqrt{(\cos 60^\circ \cos 120^\circ \cos 2\theta)^2 + (\sin 60^\circ \sin 120^\circ \sin 2\theta)^2} \\ &= \frac{2m}{n} \times \omega^2 \cdot r \sqrt{\left[\frac{1}{2} \times -\frac{1}{2} \times \cos 2\theta\right]^2 + \left[\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times \sin 2\theta\right]^2} \\ &= \frac{2m}{n} \times \omega^2 \cdot r \sqrt{\cos^2 2\theta + 9 \sin^2 2\theta} \quad \dots \text{(xvi)} \end{aligned}$$

Example 22.16. A vee-twin engine has the cylinder axes at right angles and the connecting rods operate a common crank. The reciprocating mass per cylinder is 11.5 kg and the crank radius is 75 mm. The length of the connecting rod is 0.3 m. Show that the engine may be balanced for primary forces by means of a revolving balance mass.

If the engine speed is 500 r.p.m. What is the value of maximum resultant secondary force ?

Solution. Given : $2\alpha = 90^\circ$ or $\alpha = 45^\circ$; $m = 11.5$ kg ; $r = 75$ mm = 0.075 m ; $l = 0.3$ m ; $N = 500$ r.p.m. or $\omega = 2\pi \times 500 / 60 = 52.37$ rad/s

We know that resultant primary force,

$$\begin{aligned} F_P &= 2m \cdot \omega^2 \cdot r \sqrt{(\cos^2 \alpha \cos \theta)^2 + (\sin^2 \alpha \sin \theta)^2} \\ &= 2m \cdot \omega^2 \cdot r \sqrt{(\cos^2 45^\circ \cos \theta)^2 + (\sin^2 45^\circ \sin \theta)^2} \\ &= 2m \cdot \omega^2 \cdot r \sqrt{\left[\frac{\cos \theta}{2}\right]^2 + \left[\frac{\sin \theta}{2}\right]^2} = m \cdot \omega^2 \cdot r \end{aligned}$$

Since the resultant primary force $m \cdot \omega^2 \cdot r$ is the centrifugal force of a mass m at the crank radius r when rotating at ω rad / s, therefore, the engine may be balanced by a rotating balance mass.

Maximum resultant secondary force

We know that resultant secondary force,

$$F_S = \sqrt{2} \times \frac{m}{n} \times \omega^2 \cdot r \sin 2\theta \quad \dots \text{(When } 2\alpha = 90^\circ \text{)}$$

This is maximum, when $\sin 2\theta$ is maximum i.e. when $\sin 2\theta = \pm 1$ or $\theta = 45^\circ$ or 135° .

\therefore Maximum resultant secondary force,

$$\begin{aligned} F_{S_{\max}} &= \sqrt{2} \times \frac{m}{n} \times \omega^2 \cdot r \quad \dots \text{(Substituting } \theta = 45^\circ \text{)} \\ &= \sqrt{2} \times \frac{11.5}{0.3/0.075} (52.37)^2 \cdot 0.075 = 836 \text{ N Ans.} \quad \dots (\because n = l/r) \end{aligned}$$

Example 22.17. The reciprocating mass per cylinder in a 60° V-twin engine is 1.5 kg. The stroke and connecting rod length are 100 mm and 250 mm respectively. If the engine runs at 2500 r.p.m., determine the maximum and minimum values of the primary and secondary forces. Also find out the crank position corresponding these values.

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Solution. Given $2\alpha = 60^\circ$ or $\alpha = 30^\circ$, $m = 1.5$ kg ; Stroke = 100 mm or $r = 100/2 = 50$ mm = 0.05 m ; $l = 250$ mm = 0.25 m ; $N = 250$ r.p.m. or $\omega = 2\pi \times 2500 / 60 = 261.8$ rad/s

Maximum and minimum values of primary forces

We know that the resultant primary force,

$$\begin{aligned} F_P &= 2m\omega^2 r \sqrt{(\cos^2 \alpha \cdot \cos \theta)^2 + (\sin^2 \alpha \cdot \sin \theta)^2} \\ &= 2m\omega^2 r \sqrt{(\cos^2 30^\circ \cos \theta)^2 + (\cos^2 30^\circ \sin \theta)^2} \\ &= 2m\omega^2 r \sqrt{\left(\frac{3}{4} \cos \theta\right)^2 + \left(\frac{1}{4} \sin \theta\right)^2} \\ &= \frac{m}{2} \times \omega^2 r \sqrt{9 \cos^2 \theta + \sin^2 \theta} \quad \dots(i) \end{aligned}$$

The primary force is maximum, when $\theta = 0^\circ$. Therefore substituting $\theta = 0^\circ$ in equation (i), we have maximum primary force,

$$F_{P(max)} = \frac{m}{2} \times \omega^2 r \times 3 = \frac{1.5}{2} (261.8)^2 0.05 \times 3 = 7710.7 \text{ N Ans.}$$

The primary force is minimum, when $\theta = 90^\circ$. Therefore substituting $\theta = 90^\circ$ in equation (i), we have minimum primary force,

$$F_{P(min)} = \frac{m}{2} \times \omega^2 r = \frac{1.5}{2} (261.8)^2 0.05 = 2570.2 \text{ N Ans.}$$

Maximum and minimum values of secondary forces

We know that resultant secondary force.

$$\begin{aligned} F_S &= \frac{2m}{n} \times \omega^2 r \sqrt{(\cos \alpha \cos 2\alpha \cos 2\theta)^2 + (\sin \alpha \sin 2\alpha \sin 2\theta)^2} \\ &= \frac{2m}{n} \times \omega^2 r \sqrt{(\cos 30^\circ \cos 60^\circ \cos 2\theta)^2 + (\sin 30^\circ \sin 60^\circ \sin 2\theta)^2} \\ &= \frac{2m}{n} \times \omega^2 r \sqrt{\left(\frac{\sqrt{3}}{2} \times \frac{1}{2} \cos 2\theta\right)^2 + \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} \sin 2\theta\right)^2} \\ &= \frac{\sqrt{3}}{2} \times \frac{m}{n} \times \omega^2 r \\ &= \frac{\sqrt{3}}{2} \times \frac{1.5}{0.25/0.05} (261.8)^2 0.05 \quad \dots (\because n = l/r) \\ &= 890.3 \text{ N Ans.} \end{aligned}$$

EXERCISES

1. A single cylinder horizontal engine runs at 120 r.p.m. The length of stroke is 400 mm. The mass of the revolving parts assumed concentrated at the crank pin is 100 kg and mass of the reciprocating parts is 150 kg. Determine the magnitude of the balancing mass required to be placed opposite to the crank at a radius of 150mm which is equivalent to all the revolving and 2/3rd of the reciprocating masses. If the crank turns 30° from the inner dead centre, find the magnitude of the unbalanced force due to the balancing mass. **[Ans. 212.4 kg]**
2. A single cylinder engine runs at 250 r.p.m. and has a stroke of 180 mm. The reciprocating parts has a mass of 120 kg and the revolving parts are equivalent to a mass of 70 kg at a radius of 90 mm. A mass is placed opposite to the crank at a radius of 150 mm to balance the whole of the revolving mass and two-thirds of the reciprocating mass. Determine the magnitude of the balancing mass and the resultant residual unbalance force when the crank has turned 30° from the inner dead centre, neglect the obliquity of the connecting rod. **[Ans. 90 kg ; 3.264 kN]**
3. A two cylinder uncoupled locomotive has inside cylinders 0.6 m apart. The radius of each crank is 300 mm and are at right angles. The revolving mass per cylinder is 250 kg and the reciprocating mass per cylinder is 300 kg. The whole of the revolving and two-third of the reciprocating masses are to be balanced and the balanced masses are placed, in the planes of rotation of the driving wheels, at a radius of 0.8 m. The driving wheels are 2 m in diameter and 1.5 m apart. If the speed of the engine is 80 km. p.h. ; find hammer blow, maximum variation in tractive effort and maximum swaying couple. **[Ans. 18.30 kN, 16.92 kN, 16.2 kN-m]**
4. A two cylinder uncoupled locomotive with cranks at 90° has a crank radius of 325 mm. The distance between the centres of driving wheels is 1.5 m. The pitch of cylinders is 0.6 m. The diameter of treads of driving wheels is 1.8 m. The radius of centres of gravity of balance masses is 0.65 m. The pressure due to dead load on each wheel is 40 kN. The masses of reciprocating and rotating parts per cylinder are 330 kg and 300 kg respectively. The speed of the locomotive is 60 km. p.h. find : 1. The balancing masses both in magnitude and position required to be placed in the planes of driving wheels to balance whole of the revolving and two-third of the reciprocating masses ; 2. The swaying couple ; 3.The variation in tractive force ; 4. The maximum and minimum pressure on rails ; and 5. The maximum speed at which it is possible to run the locomotive, in order that the wheels are not lifted from the rails. **[Ans. 200 kg ; 13 kN-m ; 17.34 kN ; 58.86 kN, 21.14 kN ; 87.54 km/h]**
5. Two locomotives are built with similar sets of reciprocating parts. One is an inside cylinder engine with two cylinders with centre lines at 0.6 m apart. The other is an outside cylinder with centre lines at 1.98 m apart. The distance between the driving wheel centres is 1.5 m in both the cases. The inside cylinder locomotive runs at 0.8 times the speed of the outside cylinder locomotive and the hammer blow of the inside cylinder locomotive is 1.2 times the hammer blow of the outside cylinder locomotive. If the diameter of the driving wheel of the outside cylinder locomotive is 1.98 m, calculate the diameter of the driving wheel of the inside cylinder locomotive. Compare also the variation in the swaying couples of the two engines. Assume that the same fraction of the reciprocating masses are balanced in both the cases. **[Ans. 1.184 m, 1.185]**
6. An air compressor has four vertical cylinders 1,2,3 and 4 in line and the driving cranks at 90° intervals reach their upper most positions in this order. The cranks are of 150 mm radius, the connecting rods 500 mm long and the cylinder centre line 400 mm apart. The mass of the reciprocating parts for each cylinder is 22.5 kg and the speed of rotation is 400 r.p.m. Show that there are no out-of-balance primary or secondary forces and determine the corresponding couples, indicating the positions of No. 1 crank for maximum values. The central plane of the machine may be taken as reference plane. **[Ans. Primary couple = 6.7 kN-m at 45° and 225° ; Secondary couple = 1.4 kN-m at 0° , 90° , 180° , 270°]**
7. A four cylinder engine has the two outer cranks at 120° to each other and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 400 mm, 700 mm, 700 mm and 500 mm. Find the reciprocating mass and the relative angular position for each of the inner cranks, if the engine is to be in complete primary balance. Also find the maximum

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unbalanced secondary force, if the length of each crank is 350 mm, the length of each connecting rod 1.7 m and the engine speed 500 r.p.m.

[Ans. 800 kg at 163° counter clockwise from crank 1, 830 kg at 312° counter clockwise from crank 1 ; 397.3 kN]

- 8.** The reciprocating masses of the first three cylinders of a four cylinder engine are 4.1, 6.2 and 7.4 tonnes respectively. The centre lines of the three cylinders are 5.2 m, 3.2 m and 1.2 m from the fourth cylinder. If the cranks for all the cylinders are equal, determine the reciprocating mass of the fourth cylinder and the angular position of the cranks such that the system is completely balanced for the primary force and couple.

If the cranks are 0.8 m long, the connecting rods 3.8 m, and the speed of the engine 75 r.p.m. ; find the maximum unbalanced secondary force and the crank angle at which it occurs.

[Ans. 6.19 t ; 7.5 kN, 33° clockwise from I.D.C.]

- 9.** In a four cylinder petrol engine equally spaced, the cranks, numbered from the front end are 1,2,3, and 4. The cranks 1 and 4 are in phase and 180° ahead of cranks 2 and 3. The reciprocating mass of each cylinder is 1 kg. The cranks are 50 mm radius and the connecting rod 200 mm long.

What are the resultant unbalanced forces and couples, primary and secondary, when cranks 1 and 4 are on top dead centre position ? The engine is rotating at 1500 r.p.m. in a clockwise direction when viewed from the front. Take the reference plane midway between cylinder 2 and 3.

- 10.** A four cylinder inline marine oil engine has cranks at angular displacement of 90°. The outer cranks are 3 m apart and inner cranks are 1.2 m apart. The inner cranks are placed symmetrically between the outer cranks. The length of each crank is 450 mm. If the engine runs at 90 r.p.m. and the mass of reciprocating parts for each cylinder is 900 kg, find the firing order of the cylinders for the best primary balancing force of reciprocating masses. Determine the maximum unbalanced primary couple for the best arrangement. **[Ans. 1-4-2-3 ; 45.7 kN-m]**

- 11.** In a four crank symmetrical engine, the reciprocating masses of the two outside cylinders *A* and *D* are each 600 kg and those of the two inside cylinders *B* and *C* are each 900 kg. The distance between the cylinder axes of *A* and *D* is 5.4 metres. Taking the reference line to bisect the angle between the cranks *A* and *D*, and the reference plane to bisect the distance between the cylinder axes of *A* and *D*, find the angles between the cranks and the distance between the cylinder axes of *B* and *C* for complete balance except for secondary couples.

Determine the maximum value of the unbalanced secondary couple if the length of the crank is 425 mm, length of connecting rod 1.8 m and speed is 150 r.p.m.

[Ans. A = 210°, B = 54.7°, C = 305.3°, D = 150°; 2.2 m ; 67 N-m]

- 12.** In a four cylinder inline engine, the cylinders are placed symmetrically along the longitudinal axis, with a centre distance of 2.4 m between the outside cylinders and 0.6 m between the inside cylinders. The cranks between the two inside cylinders are at 90° to each other and the mass of reciprocating parts of each of these is 225 kg. All the four cranks are of 0.3 m radius. If the system is to be completely balanced for the primary effects, determine 1. The mass of the reciprocating parts of each of the outside cranks, and 2. The angular position of the outside cranks with reference to the nearest inside cranks, measured in clockwise direction and draw an end view of the four primary cranks marking these angles therein.

With the above arrangement, evaluate the secondary unbalanced effects completely, with reference to a plane through the centre line of cylinder no. 1 and show by means of an end view the angular position of these with reference to secondary crank no. 1. The engine is running at 180 r.p.m. and the length of each connecting rod is 1.2 m.

[Ans. 164 kg each ; 128° and 148° ; 814 kN and 12.7 kN-m]

- 13.** A six-cylinder, single acting, two stroke Diesel engine is arranged with cranks at 60° for the firing sequence 1-4-5-2-3-6. The cylinders, numbered 1 to 6 in succession are pitched 1.5 m apart, except cylinders 3 and 4 which are 1.8 m apart. The reciprocating and revolving masses per line are 2.2 tonnes and 1.6 tonnes respectively. The crank length is 375 mm, the connecting rod length is 1.6 m, and the speed is 120 r.p.m.

Determine the maximum and minimum values of the primary couple due to the reciprocating and revolving parts. Also find the maximum secondary couple and angular position relative to crank No. 1. Take the plane between the cylinders 3 and 4 as the reference plane.

14. A three cylinder radial engine driven by a common crank has the cylinders spaced at 120° . The stroke is 125 mm, length of the connecting rod 225 mm and the mass of the reciprocating parts per cylinder 2 kg. Calculate the primary and secondary forces at crank shaft speed of 1200 r.p.m.
[Ans. 3000 N ; 830 N]
15. The pistons of a 60° twin V-engine has strokes of 120 mm. The connecting rods driving a common crank has a length of 200 mm. The mass of the reciprocating parts per cylinder is 1 kg and the speed of the crank shaft is 2500 r.p.m. Determine the magnitude of the primary and secondary forces.
[Ans. 6.3 kN ; 1.1 kN]
16. A twin cylinder V-engine has the cylinders set at an angle of 45° , with both pistons connected to the single crank. The crank radius is 62.5 mm and the connecting rods are 275 mm long. The reciprocating mass per line is 1.5 kg and the total rotating mass is equivalent to 2 kg at the crank radius. A balance mass fitted opposite to the crank, is equivalent to 2.25 kg at a radius of 87.5 mm. Determine for an engine speed of 1800 r.p.m. ; the maximum and minimum values of the primary and secondary forces due to the inertia of reciprocating and rotating masses.
[Ans. Primary forces : 3240 N (max.) and 1830 N (min.)
Secondary forces : 1020 N (max.) and 470 N (min.)]

DO YOU KNOW ?

- Write a short note on primary and secondary balancing.
- Explain why only a part of the unbalanced force due to reciprocating masses is balanced by revolving mass.
- Derive the following expressions, for an uncoupled two cylinder locomotive engine :
(a) Variation in tractive force ; (b) Swaying couple ; and (c) Hammer blow.
- What are in-line engines ? How are they balanced ? It is possible to balance them completely ?
- Explain the 'direct and reverse crank' method for determining unbalanced forces in radial engines.
- Discuss the balancing of V-engines.

OBJECTIVE TYPE QUESTIONS

- The primary unbalanced force is maximum when the angle of inclination of the crank with the line of stroke is
(a) 0° (b) 90° (c) 180° (d) 360°
- The partial balancing means
(a) balancing partially the revolving masses
(b) balancing partially the reciprocating masses
(c) best balancing of engines
(d) all of the above
- In order to facilitate the starting of locomotive in any position, the cranks of a locomotive, with two cylinders, are placed at to each other.
(a) 45° (b) 90° (c) 120° (d) 180°
- In a locomotive, the ratio of the connecting rod length to the crank radius is kept very large in order to
(a) minimise the effect of primary forces (b) minimise the effect of secondary forces
(c) have perfect balancing (d) start the locomotive quickly
- If c be the fraction of the reciprocating parts of mass m to be balanced per cylinder of a steam locomotive with crank radius r , angular speed ω , distance between centre lines of two cylinders a , then the magnitude of the maximum swaying couple is given by
(a) $\frac{1-c}{2} \times mr\omega^2 a$ (b) $\frac{1-c}{\sqrt{2}} \times mr\omega^2 a$
(c) $\sqrt{2}(1-c) mr\omega^2 a$ (d) none of these

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6. The swaying couple is maximum or minimum when the angle of inclination of the crank to the line of stroke (θ) is equal to
 (a) 45° and 135° (b) 90° and 135°
 (c) 135° and 225° (d) 45° and 225°
7. The tractive force is maximum or minimum when the angle of inclination of the crank to the line of stroke (θ) is equal to
 (a) 90° and 225° (b) 135° and 180° (c) 180° and 225° (d) 135° and 315°
8. The swaying couple is due to the
 (a) primary unbalanced force (b) secondary unbalanced force
 (c) two cylinders of locomotive (d) partial balancing
9. In a locomotive, the maximum magnitude of the unbalanced force along the perpendicular to the line of stroke, is known as
 (a) tractive force (b) swaying couple (c) hammer blow (d) none of these
10. The effect of hammer blow in a locomotive can be reduced by
 (a) decreasing the speed
 (b) using two or three pairs of wheels coupled together
 (c) balancing whole of the reciprocating parts
 (d) both (a) and (b)
11. Multi-cylinder engines are desirable because
 (a) only balancing problems are reduced (b) only flywheel size is reduced
 (c) both (a) and (b) (d) none of these
12. When the primary direct crank of a reciprocating engine makes an angle θ with the line of stroke, then the secondary direct crank will make an angle of with the line of stroke.
 (a) $\theta/2$ (b) θ (c) 2θ (d) 4θ
13. Secondary forces in reciprocating mass on engine frame are
 (a) of same frequency as of primary forces
 (b) twice the frequency as of primary forces
 (c) four times the frequency as of primary forces
 (d) none of the above
14. The secondary unbalanced force produced by the reciprocating parts of a certain cylinder of a given engine with crank radius r and connecting rod length l can be considered as equal to primary unbalanced force produced by the same weight having
 (a) an equivalent crank radius $r^2/4l$ and rotating at twice the speed of the engine
 (b) $r^2/4l$ as equivalent crank radius and rotating at engine speed
 (c) equivalent crank length of $r^2/4l$ and rotating at engine speed
 (d) none of the above
15. Which of the following statement is correct?
 (a) In any engine, 100% of the reciprocating masses can be balanced dynamically
 (b) In the case of balancing of multicylinder engine, the value of secondary force is higher than the value of the primary force
 (c) In the case of balancing of multimass rotating systems, dynamic balancing can be directly started without static balancing done to the system
 (d) none of the above.

ANSWERS

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (c) | 2. (b) | 3. (b) | 4. (b) | 5. (b) |
| 6. (d) | 7. (d) | 8. (a) | 9. (c) | 10. (d) |
| 11. (c) | 12. (c) | 13. (b) | 14. (a) | 15. (c) |